Principals of Design & Manufacturing Engineering
MED 111

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CONTENTS

MED 111 Principals of design & manufacturing engineering

2 Cr . hrs = { 2 lect + 1 tut +0 lab }

Mechanical Design:

Mechanical components , motion and power transmission elements , standard machine elements (threads , fasteners ,locking devices , keys , splines , gears , pulleys , bearings , pipe connections , etc. ..).

Mechanical Manufacturing:

welding and riveting conventions, basics of machine elements design , stress analysis, basic machining processes, applications of robotics technology.

CHAPTER-1

Mechanical Design

Outline the major steps involved in a mechanical design process.

Recognize the importance of mechanical design for solving the technical, global, and environmental challenges that society faces.

Recognize the importance of innovation in designing effective engineered products, systems, and processes.

Recognize the importance of multidisciplinary teams, collaboration, and technical communication in engineering.

Be familiar with some of the processes and machine tools used in manufacturing.

Understand how patents are used to protect a newly developed technology in the business side of engineering.

Describe the role played by computer-aided engineering tools in linking mechanical design, analysis, and manufacturing.

2.1 OVERVIEW

The National Academy of Engineering (NAE) has identified 14 so-called Grand Challenges facing the global engineering community and profession in the twenty-first century. These challenges are reshaping how engineers view themselves, how and what they learn, and how they think. They are also broadening the perspective of engineers and how they view the communities they impact. The 14 challenges are as follows:

- Make solar energy economical
- Provide energy from fusion
- Develop carbon sequestration methods
- Manage the nitrogen cycle
- Provide access to clean water
- Restore and improve urban infrastructure
- Advance health informatics
- Engineer better medicines
- Reverse-engineer the brain
- Prevent nuclear terror

- Secure cyberspace
- Enhance virtual reality
- Advance personalized learning
- Engineer the tools of scientific discovery

Not only will mechanical engineers play important roles in each of these challenges, but they will also take on significant technical and global leadership roles in a number of the challenges. Upon reading this list, you may even resonate with one or more of the challenges, perhaps envisioning yourself creating innovative solutions that impact millions of lives. Although the challenges span many scientific and engineering disciplines, the principle that connects them all is *design.* Many multidisciplinary teams will need to design innovative and effective solutions to meet the myriad of sub challenges that each challenge embodies. The focus of this chapter is on understanding the fundamental principles and having the skills necessary to be part of, to contribute to, or to lead a successful design process.

Element 1: Mechanical design

While discussing the differences between engineers, scientists, and mathematicians in Chapter 1, we saw that the word "engineering" is related to both "ingenious" and "devise." In fact, the process of developing something new and creative lies at the heart of the engineering profession. The ultimate goal, after all, is to build hardware that solves one of the global society's technical problems. The objective of this chapter is to introduce you to some of the issues arising when a new product is designed, manufactured, and patented. We will also explore mechanical design through case studies in the areas of small-device design, large-system design, and computer-aided engineering. The relationship of this chapter to the hierarchy of mechanical engineering disciplines is shown by the shaded boxes in Figure 2.1.

You don't need a formal education in engineering to have a good idea for a new or improved product. In fact, your interest in studying mechanical engineering may have been sparked by your own ideas for building hardware. The elements of mechanical engineering that we will examine in the remaining chapters of this book—forces in structures and machines, materials and stresses, fluids engineering, thermal and energy systems, and motion and power transmission—are intended to set in place a foundation that will enable you to bring your ideas to reality effectively and systematically. In that respect, the approach taken in this textbook is an analog of the traditional mechanical engineering curriculum: Approximation, mathematics, and science are applied to design problems to reduce trial and error. You can use the types of calculations described in Chapters 3–8 to answer many of the questions that might arise during the design process. Those calculations are not just academic exercises; rather, they will enable you to design better, smarter, and faster.

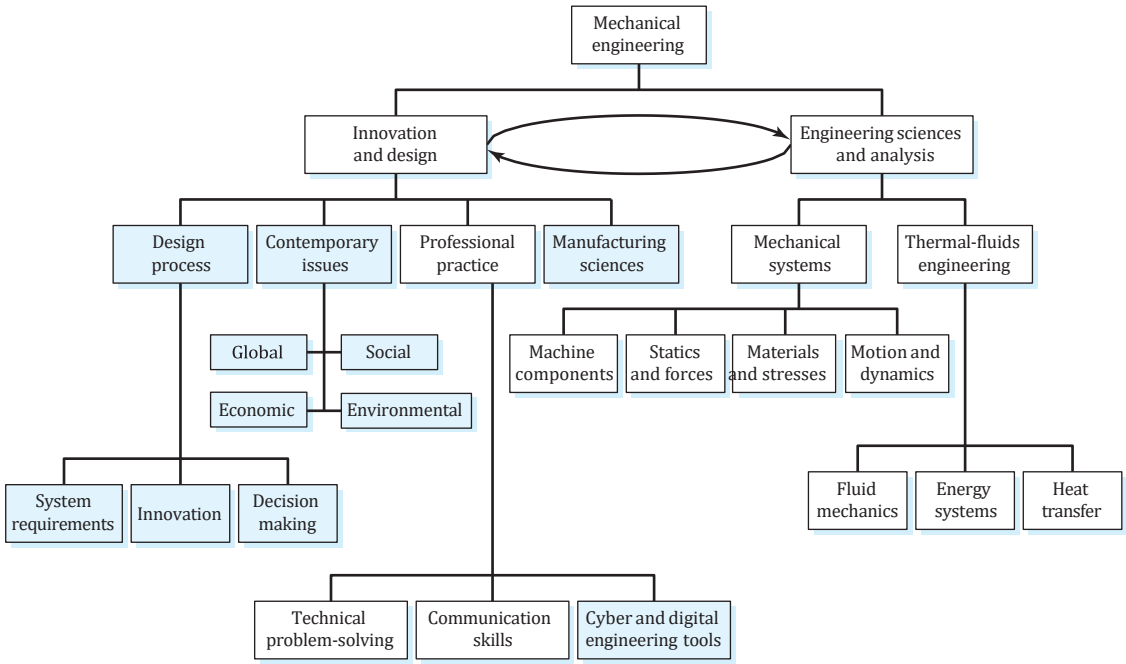


Figure 2.1 Relationship of the topics emphasized in this chapter (shaded boxes) relative to an overall program of study in mechanical engineering.

In this chapter, we will present an overview of the product development process, beginning with the definition of a design problem, proceeding to the development of a new concept, continuing to production, and culminating in the patenting of the new technology. We begin with a discussion of the high-level steps in a design process that engineers follow when they transform a new idea into reality. Once the details of the product have been determined, the hardware needs to be built economically. Mechanical engineers specify how a product will be fabricated, and Section 2.3 will introduce you to the major classes of manufacturing processes. Once the new product has been designed and built, an engineer or company will generally want to obtain a competitive advantage in the marketplace by protecting the new technology and preventing others from using it. The United States Constitution contains provisions that enable inventions to be patented, an important aspect of the business side of engineering. In the latter sections of this chapter, we will explore the design process further through case studies in the conceptual design of a spring-powered vehicle, the development of solutions to reduce the strain on urban power grids, and the use of computer-aided design tools

Perhaps you have heard someone say that engineers *discover* new technologies, much like how archaeologists discover past technologies. Although the notion of discovery drives both archaeologists and engineers, archaeologists discover what has already existed, whereas engineers discover what has never existed. However, engineers can learn a tremendous amount about design by studying existing technologies using product archaeology.

Product archaeology is the process of reconstructing the life cycle of a product—the customer requirements, design specifications, and manufacturing processes used to produce it—in order to understand the decisions that led to its development. Product archaeology was first introduced in 1998 as a way to measure the design attributes that drive cost through the analysis of the physical products themselves.[1] More recently, product archaeology has been broadened to study not only the manufacturing cost of a product, but also the global and societal context that influences its development. It also enables engineers to study the environmental impact of a product by considering the energy and material usage throughout its life cycle. When implemented in an engineering classroom, product archaeology allows students to place themselves in the minds of designers and in the time frame during which a specific product was developed in order to try to recreate the global and local conditions that led to its development.

For example, in mechanical engineering courses at Penn State University, the University at Buffalo—SUNY, and Northwestern University, students are engaged in various product archaeology projects and exercises by mimicking the process archaeologists use:

1. *Preparation:* Background research about a product, including market research, patent searches, and benchmarking existing products
2. *Excavation:* Dissecting a product, performing component analysis, creating a functional description, and reassembly of the product
3. *Evaluation:* Benchmarking existing products, conducting material and product tests
4. *Explanation:* Draw conclusions about the global, economic, environmental, and societal issues that shaped the design of the product and that currently shape the design of similar products

For example, at Penn State, students conduct "archaeological digs" of bicycles. As part of their research, dissection, and product analysis, the students unearthed the following information about bicycles that will help shape the future design of bicycles for a wide range of global markets.

Bicycles in a global context:

- Bicycles are used as ambulances in sub-Saharan Africa
- Japan has so many bicycles that they have bicycle parking structures
- In countries such as Holland, there are entire transportation infrastructures just for bicycles, including lanes, traffic signals, parking lots, road signs, and tunnels
- Many bicycles in China are electric

Bicycles in a societal context:

- A number of bicycle cafés serve organic foods and loan bikes to people to travel around town
- Henry Ford was a bike mechanic, and the Wright brothers used bicycle tubing for their first flight
- The bicycle served as a catalyst for so-called rational dress among women as part of their emancipation movement

[1]K. T. Ulrich and S. Pearson, "Assessing the Importance of Design Through Product Archaeology," *Management Science,* 1998, 44(3), 352–369.

Environmental impact of bicycles:

- There are many bike-sharing programs in European countries
- There are a wide variety of programs to encourage people to bike to work to save on carbon emissions
- Students found many statistics on bicycle commuters in U.S. cities
- They also dug up bicycle efficiency and fatality rates

Economic issues in bicycle design:

- The relative costs of bicycles versus cars including costs to make, operate, and maintain each
- The cost trade-offs with plastic bicycles versus traditional materials
- Reduced healthcare costs when bicycles are a prominent mode of transportation

One of the benefits of learning design principles using product archaeology is that it clearly demonstrates the multidisciplinary nature of product design and development. While mechanical engineers are an essential part of any product design team, products are too complex for one person to have all the necessary skills, knowledge, time, and experience. Mechanical engineers will need to interact and communicate effectively with a wide range of team members and partners, including electrical engineers, computer engineers, industrial engineers, management, marketing, physicists, chemists, material scientists, manufacturing, suppliers, and customers. (Developing the professional skills to succeed in such a dynamic and demanding multidisciplinary environment is the focus of Chapter 3.) In any product design process, the teams must:

- Develop a sense of responsibility among its members for their collective success
- Conduct debate and dialogue about various ideas and solutions while ensuring that each team member is able to contribute to the discussion
- Allow members to explain their ideas and to learn collectively
- Support decisions that are made in the best overall interest of the team, even though not all members agree
- Promote innovative solutions that address technical, global, social, environmental, and economic issues in creative and effective ways

2.2 The Design Process

In the broad view, mechanical design is the systematic process for devising a product or system that meets one of the global society's technical needs. As illustrated by the Grand Challenges (Section 2.1) and the top ten list of the mechanical engineering profession's achievements (Section 1.3), the need could lie in the area of healthcare, transportation, technology, communication, energy, or security. Engineers conceive solutions to those problems and turn their conceptions into functioning hardware.

Although a mechanical engineer might specialize in a field such as materials selection or fluids engineering, the day-to-day activities will often focus on design. In some instances, a designer will start from scratch and have the freedom to develop an original product from the concept stage onward. The technology that is developed could be so revolutionary as to

create entirely new markets and business opportunities. Smart phones and hybrid vehicles are examples of how technology is changing the way people think about communication and transportation. In other cases, an engineer's design work will be incremental and focus on improving an existing product. Examples are the addition of a high-definition video camera to a cell phone and the small yearly variations made to an automobile model.

Where does the life of a new product begin? First, a company will identify new business opportunities and define requirements for a new product, system, or service. Past, current, and potential customers are surveyed, online product reviews and feedback forums are studied, and related products are researched. Marketing, management, and engineering all provide additional input to help develop a comprehensive set of system requirements.

In the next phase, engineers exercise their creativity and develop potential concepts, select the top concept using the requirements as decision criteria, develop details (such as layout, material choice, and component sizing), and bring the hardware to production. Will the product meet the initial requirements, and can it be produced economically and safely? To answer such questions, engineers make many approximations, trade-offs, and decisions along the way. Mechanical engineers are mindful that the level of precision needed in any calculation gradually grows as the design matures from concept to final production. Resolving specific details (Will a grade 1020 steel be strong enough? What must the viscosity of the oil be? Should ball or tapered roller bearings be used?) doesn't make much sense until the design gets reasonably to its final form. After all, in the early stages of a design, the specifications for the product's size, weight, power, or performance could change. Design engineers learn to be comfortable with order-of-magnitude calculations (Section 3.6), and they are able to develop products even in the presence of ambiguity and requirements that might change over time.

Many people think that people are simply born innovative or not. Although some people are stronger right-brain thinkers, everyone can learn to become more innovative. Innovation, a familiar concept to industrial designers, artists, and marketers, is becoming a critical topic in the development of strategies around the world to solve complex social, environmental, civic, economic, and technical challenges. Initiatives centered on technology and scientific innovation are underway across the globe.

- President Obama has developed "A Strategy for American Innovation," which includes an Office of Innovation and Entrepreneurship and the formation of a National Advisory Council on Innovation and Entrepreneurship
- For the first time in history, the United States government has a chief technology officer
- The China Standards and Innovation Policy

initiative aims to analyze relationships

between standards and innovation in order to better inform global leaders

- In Australia, the minister for Innovation, Industry, Science and Research developed, for the entire nation, a Framework of Principles for Innovation Initiatives
- The Department of Science and Technology within the government of India has developed the India Innovation Initiative (i3) to create an innovation network, encouraging and promoting innovators and commercialization across the country
- The Agricultural Innovation in Africa project, funded by the Bill and Melinda Gates Foundation, is supporting efforts that contribute to agricultural scientific innovations and to technology policy improvement through Africa's Regional Economic Communities

Not only are innovation initiatives underway at the national level, but many companies have developed innovation centers to drive new product, process, and service development. Companies like Microsoft, Procter & Gamble, Accenture, IBM, AT&T, Computer Sciences Corporation, Qualcomm, and Verizon have all opened innovation centers focused on developing key scientific and technological innovations.

Mechanical engineers play significant roles in these corporate and national innovation initiatives. Recognizing and understanding how mechanical engineering design impacts the success of innovative technologies will be vital to solving the Grand Challenges. You will encounter design again in your curriculum, but you now must understand how innovation can develop a wide range of technologies to provide better engineered solutions. Figure 2.2 shows a 2 3 2 chart with Style (low/high) on the vertical axis and Technology (low/high) on the horizontal axis. This chart provides a framework to strategically develop innovative products for a wide range of customers.

In each quadrant is a different digital music player. In the lower left, the Low-Style/Low-Tech version is a standard, affordable player, designed for customers who want just to play

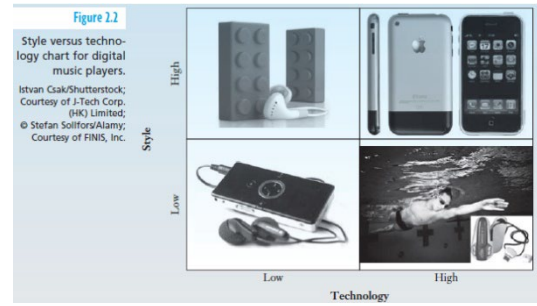


Figure 2.2
Style versus technology chart for digital music players.
Istvan Csak/Shutterstock; Courtesy of J-Tech Corp. (HK) Limited; © Stefan Sollfors/Alamy; Courtesy of FINIS, Inc.

music. The player, while not the most stylish or high-tech, provides solid, expected playback of digital music. In the lower right, the Low-Style/High-Tech version is the SwiMP3 player from FINIS. This player integrates waterproof technologies with revolutionary bone conduction of sound to provide swimmers with clear sound underwater. While the player is functionally effective, it does not need to be high in style for its intended market. In the upper left, the High-Style/Low-Tech version is a standard player shaped like a Lego® block, designed for customers who are very style conscious. In the upper right, the High-Style/High-Tech version is an Apple™ iPhone, for customers who want the latest technologies along with stylish features.

In Figure 2.3, a similar chart is shown for water purification products. In the lower left, the Low-Style/Low-Tech product is a pot used to boil water in order to eliminate micro-organisms using basic heat technology. In the lower right, the Low-Style/High-Tech product is the LIFESAVER® bottle that uses advanced nano technology to filter out even the smallest bacteria, viruses, cysts, parasites, fungi, and all other microbiological waterborne pathogens. In the upper left, the High-Style/Low-Tech product is a Fashion Bottle filtration system from Clear2O®. In the upper right, the High-Style/High-Tech product is the Hague WaterMax® system, a custom-designed water treatment system for an entire house.

Developing technically effective, consumer-safe, globally aware, and environmentally friendly products to meet a wide range of market, social, and cultural demands requires mechanical engineers who can think innovatively. Regardless of your current ability to innovate, the bottom line is that you can always become more innovative. Effective design through innovation is one of the skills that mechanical engineers will be expected to have upon graduation.

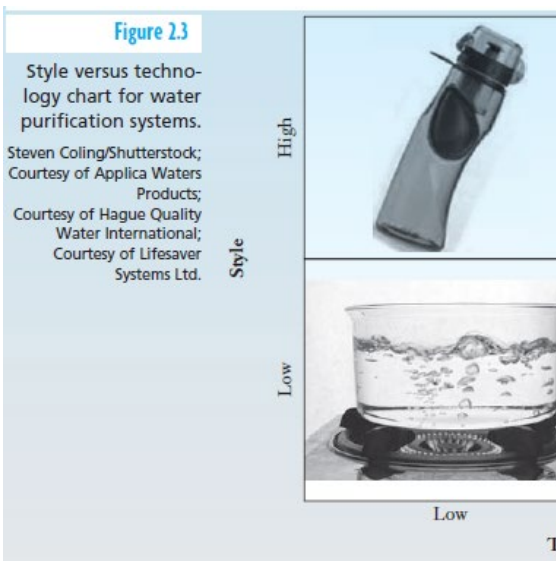


Figure 2.3

Style versus technology chart for water purification systems.

Steven Coling/Shutterstock; Courtesy of Applica Waters Products; Courtesy of Hague Quality Water International; Courtesy of Lifesaver Systems Ltd.

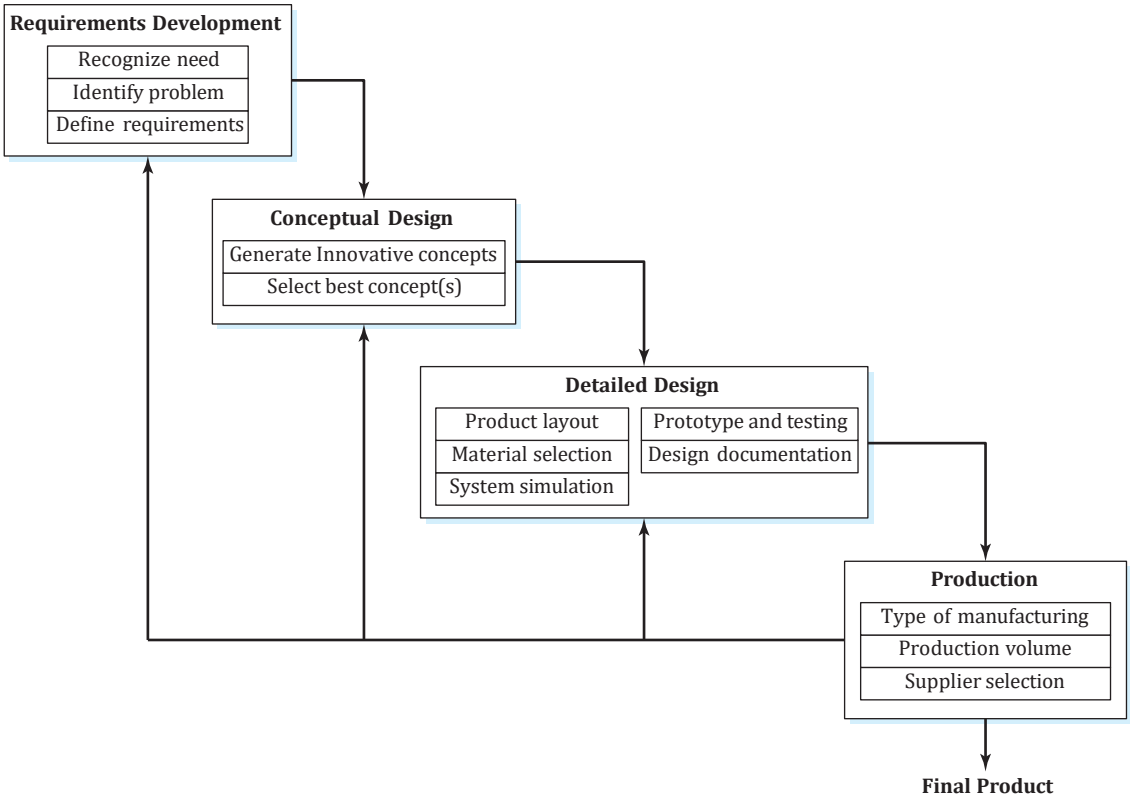


Figure 2.4 Flowchart of the prototypical mechanical design process.

From a macroscopic perspective, the mechanical design process can be broken down into four major stages, which are outlined with greater detail in Figure 2.4:

- Requirements development
- Conceptual design
- Detailed design
- Production

Requirements Development

Engineering design begins when a basic need has been identified. This could be a technical need from a certain market or a basic human need like clean water, renewable energy, or protection from natural disasters. Initially, a design engineer develops a comprehensive set of system requirements considering the following issues:

- *Functional performance*: What the product must accomplish
- *Environmental impact*: During production, use, and retirement

- *Manufacturing*: Resource and material limitations
- *Economic issues*: Budget, cost, price, profit
- *Ergonomic concerns*: Human factors, aesthetics, ease of use
- *Global issues*: International markets, needs, and opportunities
- *Life cycle issues*: Use, maintenance, planned obsolescence
- *Social factors*: Civic, urban, cultural issues

These requirements essentially represent the constraints that the design must eventually satisfy. To develop these requirements, engineers conduct extensive research and gather background information from a diverse set of sources. As mentioned in Section 1.4, engineers need to be able to communicate with a wide array of stakeholders in a design process because they will read patents that have been issued for related technologies, consult with suppliers of components that might be used in the product, attend trade shows, present product proposals to management, and meet with potential customers.

Conceptual Design

In this stage, design engineers collaboratively and creatively generate a wide range of potential solutions to the problem at hand and then select the most promising one(s) to develop. Initially, as shown in Figure 2.5, the process is guided by *divergent thinking*—a diverse set of creative ideas is developed. Some people think that creativity is reserved for artists, who are born with the ability to be innovative, and that engineers, need to be practical, leaving the creative tasks to others. The reality is that being creative is a critical part of an engineer's job; product design requires engineers who are part rational scientists and part innovative artists. Engineers can learn to be more creative, giving themselves a necessary skill set to survive in their academic and workplace careers. Many times the most creative solutions come from a

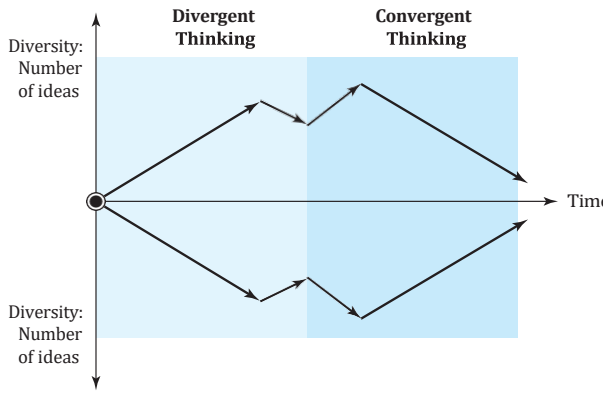


Figure 2.5
The generation and selection of ideas in conceptual design.

collaborative innovation session where people can discuss ideas with others from varied backgrounds—different professions, industries, ages, educations, cultures, and nationalities.

Once a rich set of concepts has been generated, the process is guided by *convergent thinking,* as engineers begin to eliminate ideas and converge on the best few concepts. The requirements list from the first stage is used to eliminate infeasible or inferior designs and to identify the concepts with the most potential to satisfy the requirements. These evaluations can be performed using a list of pros and cons or a matrix with ratings of the concepts using preliminary calculations to compare key requirements. Computer models and prototype hardware might also be produced at this stage to help with the selection process. At this stage, the design remains relatively fluid and changes can be made inexpensively, but the further along a product is in the development process, the more difficult and expensive changes become. This stage culminates in identifying the most promising design concept.

Detailed Design

At this point in the design process, the team has defined, innovated, analyzed, and converged its way to the best concept. However, many design and manufacturing details remain open, and each must be resolved before the product hardware can be produced. In the detailed design of the product, a number of issues must be determined:

- Developing product layout and configuration
- Selecting materials for each component
- Addressing design-for-X issues (e.g., design for reliability, manufacturing, assembly, variation, costing, recycling)
- Optimizing the final geometry, including appropriate tolerances
- Developing completed digital models of all components and assemblies
- Simulating the system using digital and mathematical models
- Prototyping and testing critical components and modules
- Developing the production plans

Simplicity

An important general principle in the detailed design stage is *simplicity.* The simpler design concept is better than a complex one, because fewer things can go wrong. Think of the most successfully engineered products, and many times it is characterized by an effective integration of design innovation, sound engineering, and functional simplicity. Keeping things as simple as possible has a well earned reputation among engineers.

Iteration

In addition, engineers need to be comfortable with the concept of iteration in a design process. *Iteration* is the process of making repeated changes and modifications to a design to improve and perfect it. For instance, if none of the generated concepts satisfactorily meet the requirements, then engineers must either revisit the requirements list or return to the concept ideation

stage. Similarly, if the production plan of the final design is infeasible, then engineers must revisit the design details and choose different materials, new configurations, or some other design detail. With each iteration, the design gradually improves—performing better, more efficiently, and more elegantly. Iteration enables you to turn hardware that works into hardware that works well.

Although engineers clearly are concerned with a design's technical aspects (forces, materials, fluids, energy, and motion), they also recognize the importance of a product's appearance, ergonomics, and aesthetics. Whether it is a consumer electronics product, the control room of an electrical power plant, or the flight deck of a commercial jetliner, the interface between the user and the hardware should be comfortable, simple, and intuitive. The *usability* of a product can become particularly problematic as its technology becomes more sophisticated. No matter how impressive the technology may be, if it is difficult to operate, customers will not embrace it as enthusiastically as they may have otherwise. In this regard, engineers often collaborate with industrial designers and psychologists to improve the appeal and usability of their products. In the end, engineering is a business venture that meets the needs of its customers.

Usability

Engineers must be very diligent in documenting the design process's engineering drawings, meeting minutes, and written reports so that others can understand the reasons behind each of the decisions. Such *documentation* is also useful for future design teams who will want to learn from and build on the present team's experiences. A design notebook (see Section 3.7) is an effective way to capture the information and knowledge created during a design process.

Documentation

Design notebooks—preferably bound, numbered, dated, and even witnessed—also support the patenting of important new technology that a company wants to prevent others from using. Drawings, calculations, photographs, test data, and a listing of the dates on which important milestones were reached are important to capture accurately how, when, and by whom the invention was developed. *Patents* are a key aspect of the business side of engineering because they provide legal protection for the inventors of new technology. Patents are one aspect of intellectual property (a field that also encompasses copyrights, trademarks, and trade secrets), and they are a right to property, analogous to the deed for a building or a parcel of land.

Patents

Patents are granted for a new and useful process, machine, article of manufacture, or composition of matter or for an improvement of them. Patents are agreements between an inventor and a national government. An inventor is granted the legal right to exclude others from making, using, offering for sale, selling, or importing an invention. In exchange, the inventor agrees to disclose and explain the invention to the general public in the written document called a *patent*. A patent is a monopoly on the new technology that expires after a certain number of years, whose duration depends on the type of patent issued and the nation issuing it. It could be

argued that the benefits of the patent system have formed the economic foundation on which our society has made its technological progress. Patents stimulate corporate research and product development because they provide a financial incentive (a limited monopoly) for innovation. By being creative, an inventor can use the protection offered by a patent to obtain an advantage over business competitors.

The United States Constitution provides the Congress with the authority to enact patent laws. Interestingly, this authority is mentioned before other (perhaps more well-known) powers of the Congress, such as declaring war and maintaining an army.

There are three primary types of patents in the United States: plant, design, and utility. As the name implies, a *plant patent* is issued for certain types of asexually reproduced plants, and it is not commonly encountered by mechanical engineers.

Design patent

A *design patent* is directed at a new, original, and ornamental design. A design patent is intended to protect an aesthetically appealing product that is the result of artistic skill; it does not protect the product's functional characteristics. For instance, a design patent could protect the shape of an automobile's body if it is attractive, pleasing to look at, or gives the vehicle a sporty appearance. However, the design patent would not protect the functional characteristic of the body, such as reducing wind drag or offering improved crash protection.

Utility patent

More commonly encountered in mechanical engineering, the *utility patent* protects the function of an apparatus, process, product, or composition of matter. The utility patent generally contains three main components:

- The *specification* is a written description of the purpose, construction, and operation of the invention
- The *drawings* show one or more versions of the invention
- The *claims* explain in precise language the specific features that the patent protects

The description provided in the patent must be detailed enough to teach someone else how to use the invention. Utility patents become valid on the date the patent is granted, and recently issued ones expire 20 years after the date of the application, which must be filed within one year of the inventor's having publicly disclosed or used the invention (e.g., by selling or offering to sell it to others, by demonstrating it at an industrial trade show, or by publishing an article on it).

To apply for a patent, engineers normally work with patent attorneys who conduct a search of existing patents, prepare the application, and interact with a national trademark and patent office. In 2009, the United States alone granted over 190,000 patents, only half of which have U.S. origins. The following shows the top ten countries ranked by the number of patents granted in the United States in 2009.

Country	Number of Patents Granted in the United States	Percentage Increase from 1999
Japan	38,066	17%
Germany	10,353	5%
South Korea	9566	160%
Taiwan	7781	72%
Canada	4393	19%
United Kingdom	4011	3%
France	3805	27%
China	2270	2193%
Italy	1837	9%
Netherlands	1558	17%

The following instead shows the top ten countries ranked by the percentage increase of patents granted in the United States between 2009 and 1999.

Country	Number of Patents Granted in the United States	Percentage Increase from 1999
China	2270	2193%
India	720	532%
Malaysia	181	432%
Singapore	493	224%
South Korea	9566	160%
Poland	43	115%
Israel	1525	93%
Ireland	189	89%
Australia	1550	86%
Taiwan	7781	72%

While obtaining a patent from a particular country protects an individual or company in that country, sometimes international patent protection is preferred. The World Intellectual Property Organization (WIPO – www.wipo.int) offers individual and corporate patent applicants a way to obtain patent protection internationally. In 2009, 155,900 patent applications from all over the world were filed with the WIPO.

Sometimes engineers want a quick prototype to finalize some product features in preparation for a patent application, for product documentation, or to communicate product details to others. A picture may be worth a thousand words, but a physical prototype is often useful for engineers to visualize complex machine components. Many times these prototypes can be physically tested

Figure 2.6

Three-dimensional printing systems and representative prototyped products.

Courtesy of Stratasys, Inc; Courtesy of Z Corporation.



Rapid prototyping

so that trade-off decisions are made based on the results of measurements and analysis. One method for producing such components is called *rapid prototyping*, and its key capability is that complex three-dimensional objects are fabricated directly from a computer-generated drawing, often in a matter of hours.

Some rapid prototyping systems use lasers to fuse layers of a liquid polymer together (a process known as *stereolithography*) or to fuse raw material in powder form. Another prototyping technique moves a printhead (similar to that used in an ink-jet printer) to spray a liquid adhesive onto a powder and "glue-up" a prototype bit by bit. In essence, the rapid prototyping system is a three-dimensional printer capable of transforming an electronic representation of the component into plastic, ceramic, or metallic parts. Figure 2.6 depicts two 3-D printing rapid prototyping systems and representative products that are created. These rapid prototyping technologies can produce durable and fully functional prototypes that are fabricated from polymers and other materials. The components can be assembled, tested, and even sometimes used as production parts.

Production

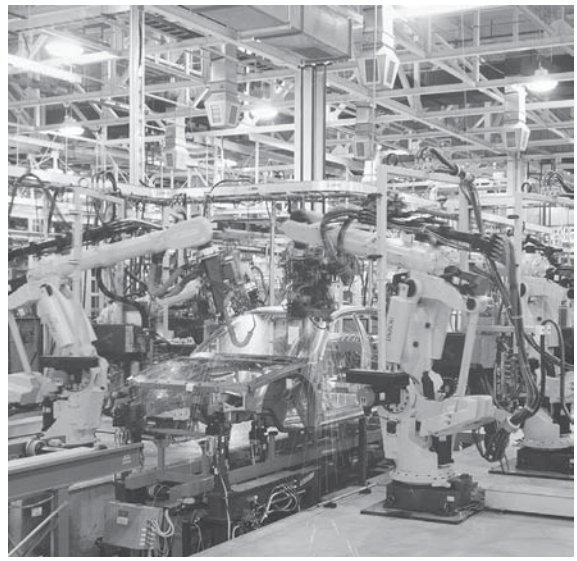
The engineer's involvement is not over once the working prototype has been delivered and the finishing touches have been placed on the drawings. Mechanical engineers work in a broad environment, and their designs are viewed with a critical eye beyond the criterion of simply whether the solution functions as intended. After all, if the product is technically superb but requires expensive materials and manufacturing operations, customers might avoid the product and select one that is more balanced in its cost and performance.

Therefore, even at the requirements development stage, engineers must take into account manufacturing requirements for the production stage. After all, if you're going to take the time to design something, it had better be something that actually can be built, preferably at a low cost. The materials selected for a product influence how it can be manufactured. A part that is machined from metal might be best for one design concept, but a plastic component produced by injection molding might be the better choice for another. In the end, the design's function, shape, materials, cost, and manner of production are tightly interconnected and balanced throughout the design process.

Figure 2.7

Robots automate welding on a mass-production line for automobile frames.

© 2001 General Motors Corporation. Used with permission of GM Media Archives.



Once the detailed design has been completed, a designer becomes involved with the fabrication and production of the product. In part, the fabrication techniques that an engineer selects depend on the time and expense of setting up the tooling and machines necessary for production. Some systems—for instance, automobiles, air conditioners, microprocessors, hydraulic valves, and computer hard disk drives—are mass-produced, a term that denotes the widespread use of mechanical automation. As an example, Figure 2.7 shows an assembly line where robots weld frames in an automotive manufacturing plant. Historically, these kinds of assembly lines comprised custom tools and specialized fixtures that were capable of efficiently producing only certain types of components for certain types of vehicles. But now flexible manufacturing systems allow a production line to quickly reconfigure for different components of different vehicles. Because in *mass production* finished products can be produced relatively quickly, a company can cost-effectively allocate a large amount of factory floor space, and many expensive machine tools, even though any one of them might perform only simple tasks, like drilling a few holes or polishing a single surface.

Mass production

Aside from hardware produced by means of mass manufacturing, other products (such as commercial jetliners) are made in relatively small quantities, or they are unique (such as the Hubble Space Telescope). Some one-of-a-kind products can even be produced directly from a computer-generated drawing by using what is, in essence, a three-dimensional printer (Figure 2.6). The best production method for a product depends on the quantity to be produced, the allowable cost, and the level of precision necessary. In the next section, we review the most prominent production and manufacturing methods.

pRoToTYpIng

Although manufacturing and rapid prototyping technologies continue to advance, virtual prototyping is gaining acceptance as an effective decision support tool in engineering design. Virtual prototyping takes advantage of advanced visualization and simulation technologies available in the fields of virtual reality, scientific visualization, and computer-aided design to provide realistic digital representations of components, modules, and products, as shown in the C6 environment at Iowa State in Figure 2.8. Virtual prototyping also capitalizes on advanced hardware that provides haptic (tactile) feedback to an engineer using forces, vibrations, and motions, as shown with the PHANTOM® six-degrees-of-freedom interface from SensAble Technologies in use at Boeing in Figure 2.9 (see on page 50).

Virtual prototyping allows for new prototypes to be created quickly at a potentially greatly reduced cost. When used in design processes, virtual prototyping facilitates the implementation of many iterations because design changes can be made rapidly on the digital model. Also, 3-D scanners are allowing digital models to be created quickly and accurately from physical models.

Another growing trend in mechanical engineering is *augmented reality* where virtual prototyping is blended with actual system operation to create a hybrid simulation environment. Mechanical engineers are integral parts of many applications of augmented reality, including robotic surgery assist systems such as the da Vinci™ and RoSS (Robotic Surgical Simulator) systems, and motion simulation environments such as the ones shown in Figure 2.10 (see on page 50).

Figure 2.8

An immersive virtual reality environment at Iowa State University.

MetaBlast educational videogame courtesy of VRAC and Iowa State University.

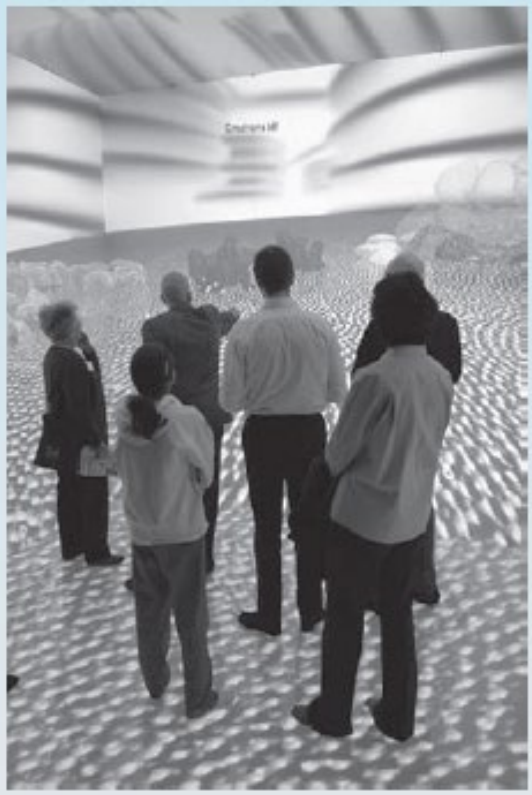


Figure 2.9

A Boeing engineer using a PHANTOM® interface to make more informed design decisions.

Courtesy of Boeing.

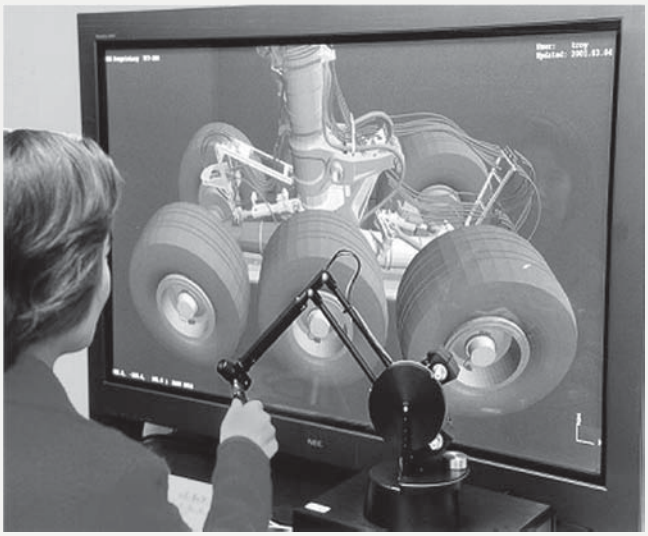








Figure 2.10 Augmented reality in motion simulation at the U.S. Air Force, University of Iowa, and the University at Buffalo–SUNY.

Photo by Javier Garcia courtesy of US Air Force; Courtesy of the National Advanced Driving Simulator at the University of Iowa; © 2011 University at Buffalo/Douglas Levere.

2.3 Manufacturing Processes

Manufacturing technologies are so economically important because they are the means for adding value to raw materials by converting them into useful products. Of the many different manufacturing processes, each is well suited to a particular need based on environmental impact, dimensional accuracy, material properties, and the mechanical component's shape. Engineers select processes, identify the machines and tools, and monitor production to ensure that the final product meets

its specifications. The main classes of manufacturing processes are as follows:

- *Casting* is the process whereby liquid metal, such as gray iron, aluminum, or bronze, is poured into a mold, cooled, and solidified.
- *Forming* encompasses a family of techniques whereby a raw material is shaped by stretching, bending, or compression. Large forces are applied to plastically deform a material into its new permanent shape.
- *Machining* refers to processes where a sharp metal tool removes material by cutting it. The most common machining methods are drilling, sawing, milling, and turning.
- *Joining* operations are used to assemble subcomponents into a final product by welding, soldering, riveting, bolting, or adhesively bonding them. Many bicycle frames, for instance, are welded together from individual pieces of metal tubing.
- *Finishing* steps are taken to treat a component's surface to make it harder, improve its appearance, or protect it from the environment. Some processes include polishing, electroplating, anodizing, and painting.

In the remainder of this section, we describe the processes of casting, forming, and machining in additional detail.

Casting

In *casting,* liquid metal is poured into the cavity of a mold, which can be expendable or reusable. The liquid then cools into a solid object with the same shape as the mold. An attractive feature of casting is that complex shapes can be produced as solid objects without the need to join any pieces. Casting is an efficient process for creating many copies of a three-dimensional object, and, for that reason, cast components are relatively inexpensive. On the other hand, defects can arise if the metal solidifies too soon and prevents the mold from filling completely. The surface finish of cast components generally has a rough texture, and they might require additional machining operations to produce smooth and flat surfaces. Some examples of cast components include automotive engine blocks, cylinder heads, and brake rotors and drums (Figure 2.11, see on page 52).

Rolling

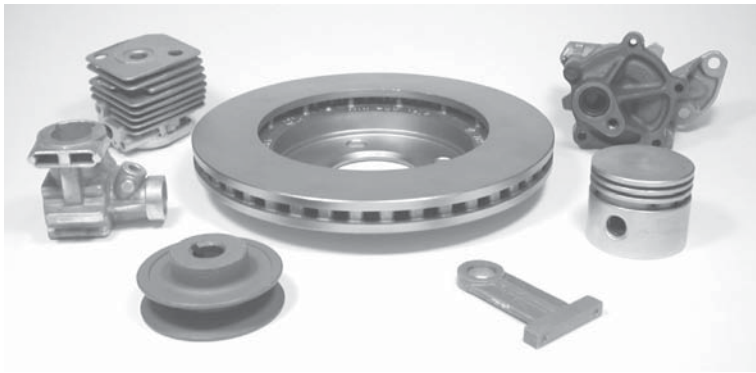
One kind of a forming operation is called *rolling,* which is the process of reducing the thickness of a flat sheet of material by compressing it between rollers, not unlike making cookie or pizza dough. Sheet metal that is produced in this manner is used to make aircraft wings and fuselages, beverage containers, and the body panels of automobiles. *Forging* is another forming process, and it is based on the principle of heating, impacting, and plastically deforming metal into a final shape. Industrial-scale forging is the modern version of the blacksmith's art of working metal by hitting it with a hammer against an anvil. Components that are produced by forging include some

Forging

Figure 2.11

Examples of hardware produced by casting: a disk-brake rotor, automotive-oil pump, piston, bearing mount, V-belt sheave, model-airplane engine block, and a two-stroke engine cylinder.

Image courtesy of the authors.



crankshafts and connecting rods in internal combustion engines. Compared to castings, a forged component is strong and hard, and for that reason, many hand tools are produced this way (Figure 2.12).

Extrusion

The forming process known as *extrusion* is used to create long straight metal parts whose cross sections may be round, rectangular, L-, T-, or C-shaped, for instance. In extrusion, a mechanical or hydraulic press is used to force heated metal through a tool (called a die) that has a tapered hole ending in the shape of the finished part's cross section. The die is used to shape the raw material, and it is made from a metal that is much harder than what is being formed. Conceptually, the process of extrusion is not unlike the familiar experience of squeezing toothpaste out of a tube. Figure 2.13 shows examples of aluminum extrusions with a variety of cross sections.

Machining

Machining refers to processes whereby material is gradually removed from a workpiece in the form of small chips. The most common machining methods are called drilling, sawing, milling, and turning. Machining operations are capable of producing mechanical components with dimensions and shapes that are far more precise than their cast or forged counterparts. One drawback of machining is that (by its very nature) the removed material

Figure 2.12

Examples of hardware produced by forging.

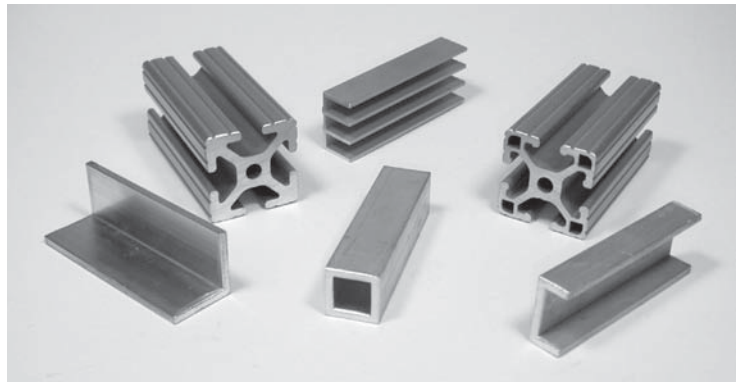
Image courtesy of the authors.



Figure 2.13

Examples of aluminum extrusions.

Image courtesy of the authors.



is wasted. In a production line, machining operations are often combined with casting and forging when cast or forged components require additional operations to flatten surfaces, make holes, and cut threads (Figure 2.14).

Drill press

Machining tools include drill presses, bandsaws, lathes, and milling machines. Each tool is based on the principle of removing unwanted material from a workpiece by means of the cutting action of sharpened blades. The *drill press* shown in Figure 2.15 (see on page 54) is used to bore round holes into a workpiece. A drill bit is held in the rotating chuck, and, as a machinist turns the pilot wheel, the bit is lowered into the workpiece's surface. As should be the case whenever metal is machined, the point where the bit cuts into the workpiece is lubricated. The oil reduces friction and helps remove heat from the cutting region. For safety reasons, vises and clamps are used to hold the workpiece securely and to prevent material from shifting unexpectedly.

Band saw

A machinist uses a *band saw* to make rough cuts through metal (Figure 2.16, see on page 54). The blade is a long, continuous loop with sharp teeth on one edge, and it rides on the drive and idler wheels. A variable-speed motor enables the operator to adjust the blade's speed depending on the type and thickness of the material being cut. The workpiece is supported on a table that is capable of being tilted for cuts that are to be made at an angle.

Figure 2.14

This body for a hydraulic valve assembly was first cast from aluminum (left) and then machined in order to produce holes, flatten surfaces, and cut threads (right).

Image courtesy of the authors.

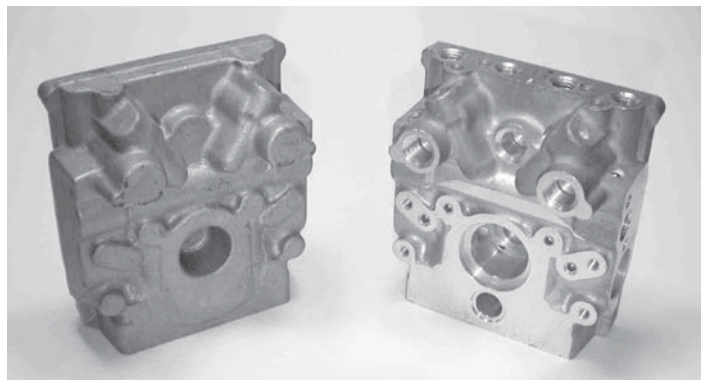


Figure 2.15

(a) Major components of a drill press.

© David J. Green-engineering themes/Alamy.

(b) Different types of holes that can be produced.

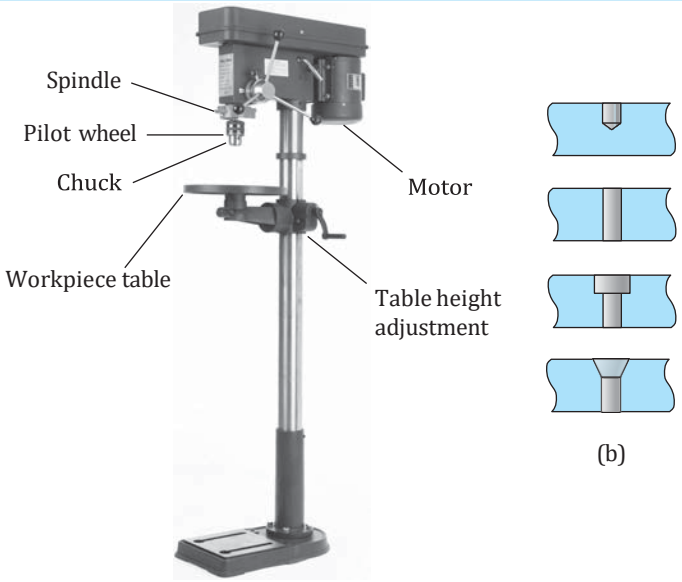


Figure 2.16

Major elements of a bandsaw.

Image courtesy of the authors.

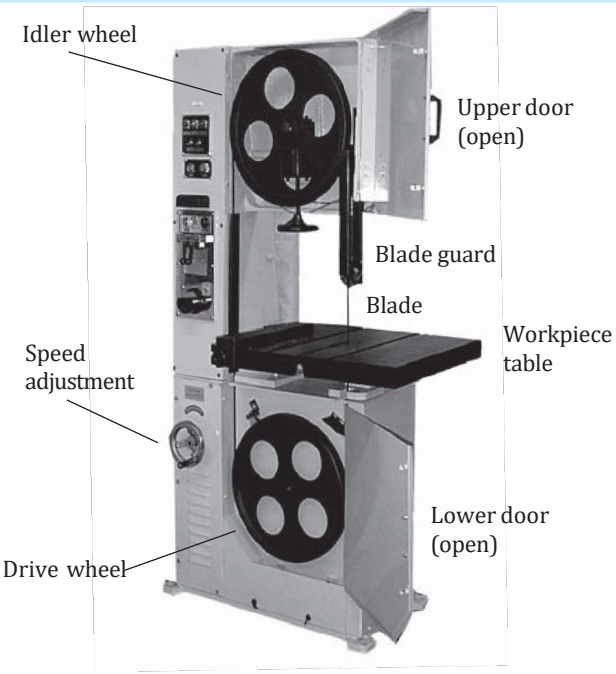
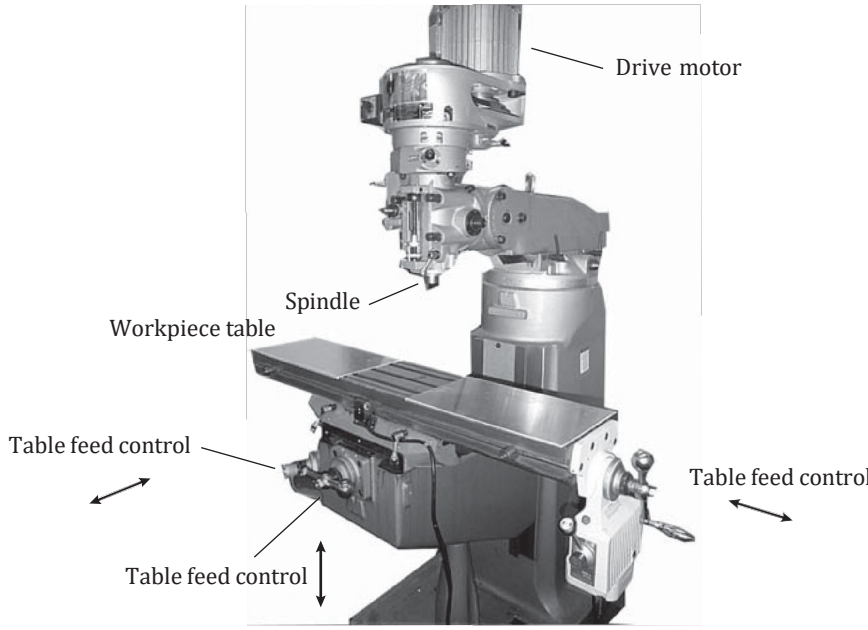


Figure 2.17

Major elements of a milling machine.

Image courtesy of the authors.



The machinist feeds the workpiece into the blade and guides it by hand to make straight or slightly curved cuts. When the blade becomes dull and needs to be replaced or if it breaks, the bandsaw's internal blade grinder and welder are used to clean up the blade's ends, connect them, and reform a loop.

Milling machine

A *milling machine* (or *mill*) is used for machining the rough surfaces of a workpiece flat and smooth and for cutting slots, grooves, and holes (Figure 2.17). The milling machine is a versatile machine tool in which the workpiece is moved slowly relative to a rotating cutting tool. The workpiece is held by a vise on an adjustable table so that the part can be accurately moved in three directions (along the plane of the table and perpendicular to it) to locate the workpiece precisely beneath the cutting bit. A piece of metal plate might be cut first to an approximate shape with a bandsaw, and then the milling machine could be used to form the surfaces smooth and the edges square to their final dimensions.

Lathe

A machinist's *lathe* holds a workpiece and rotates it about the centerline as a sharpened tool removes chips of material. The lathe is therefore used to produce cylindrical shapes and other components that have an axis of symmetry. Some applications for using a lathe are the production of shafts and the resurfacing of disk brake rotors. A lathe can be used to reduce the diameter of a shaft by moving the cutting tool along the shaft's length as it rotates. Threads, shoulders that locate bearings on a shaft, and grooves for holding retaining clips can be made in this manner.

Machine tools can be controlled by hand operation or by a computer. Computer-aided manufacturing uses computers to control machine tools to cut and shape metals and other material with remarkable precision.

Figure 2.18

A numerically controlled milling machine can produce hardware directly from instructions created by 3-D CAD software packages.

Fotosearch.



Numerical control

Machining operations are controlled by a computer when a mechanical component is particularly complicated, when high precision is required, or when a repetitive task must be performed quickly on a large number of parts. In those cases, a *numerically controlled* machine tool is capable of faster and more precise machining than a human operator. Figure 2.18 shows an example of a numerically controlled milling machine. This mill can perform the same types of operations as a conventional one. However, instead of being manually operated, it is programmed either through entries on a keypad or by downloading instructions created by computer-aided engineering software. Computer-controlled machine tools offer the potential to produce physical hardware seamlessly from a computer-generated drawing. With the ability to quickly reprogram machine tools, even a small general-purpose shop can produce high-quality machined components. In Section 2.6, we explore a case study in which digital computer-aided design (CAD) product models are used to design, analyze, and manufacture a mechanical component used in the medical industry.

Customized production

Some of the same technologies used to create rapid product prototypes for design evaluation are beginning to be used for *custom production*. Rapid or direct digital manufacturing is a class of additive fabrication techniques to produce custom or replacement parts. A mass-manufacturing line takes advantage of mechanical automation, but those systems are intended to produce many identical parts. Rapid manufacturing systems take precisely the opposite viewpoint: One-of-a-kind components are produced directly from a computer-generated electronic file. Electronic representation can be produced by using computer-aided design software or by scanning a physical object. This capability offers the potential for creating complex customized products at a reasonable cost. While rapid prototyping typically uses thermoplastics, photopolymers, or ceramics to create the parts, rapid manufacturing technologies can now also use a variety of metals and alloys. This allows engineers to create fully functional parts extremely quickly.

Table 3.4 Base Units in the USCS

Quantity	USCS Base Unit	Abbreviation
Length	foot	ft
Force	pound	lb
Time	second	s
Electric current	ampere	A
Temperature	degree Rankine	°R
Amount of substance	mole	mol
Light intensity	candela	cd

shorter terminology "pound" and the abbreviation "lb" are more common, and that convention is used throughout this textbook.

Pound-mass
Slug

Another distinction between the USCS and SI is that the USCS employs two different dimensions for mass: the *pound-mass* and the *slug*. The pound-mass unit is abbreviated lbm. There is no abbreviation for the slug, and so the full name is written out adjacent to a numerical value. It is also conventional to use the plural "slugs." This dimension's name appears to have been chosen historically to refer to a chunk or lump of material, and it is unrelated to the small land mollusk of the same name. In mechanical engineering, the slug is the preferred unit for calculations involving such quantities as gravitation, motion, momentum, kinetic energy, and acceleration. However, the pound-mass is a more convenient dimension for engineering calculations involving the material, thermal, or combustion properties of liquids, gases, and fuels. Both dimensions for mass will be used in their conventional mechanical engineering contexts throughout this textbook.

In the final analysis, however, the slug and pound-mass are simply two different derived units for mass. Because they are measures of the same physical quantity, they are also closely related to one another. In terms of the USCS's base units of pounds, seconds, and feet, the slug is defined as follows:

$$1 \text{ slug} = 1 \frac{\text{lb ? s}^2}{\text{ft}} \tag{3.2}$$

Referring to the second law of motion, one pound of force will accelerate a one-slug object at the rate of one foot per second per second:

$$1 \text{ lb} = (1 \text{ slug}) \left(1 \frac{\text{ft}}{\text{s}^2}\right) = 1 \frac{\text{slug ? ft}}{\text{s}^2} \tag{3.3}$$

On the other hand, the pound-mass is defined as the quantity of mass that weighs one pound. One pound-mass will accelerate at the rate of 32.174 ft/s^2 when one pound of force acts on it:

$$1 \text{ lbm} = \frac{1 \text{ lb}}{32.174 \text{ ft/s}^2} = 3.1081 \times 10^{-2} \frac{\text{lb ? s}^2}{\text{ft}} \tag{3.4}$$

The numerical value of 32.174 ft/s^2 is taken as the reference acceleration because it is the Earth's gravitational acceleration constant. By comparing

Equations (3.2) and (3.4), we see that the slug and pound-mass are related by

$$1 \text{ slug} = 32.174 \text{ lbm} \qquad 1 \text{ lbm} = 3.1081 \times 10^{22} \text{ slugs} \tag{3.5}$$

In short, the slug and pound-mass are both defined in terms of the action of a one-pound force, but the reference acceleration for the slug is 1 ft/s^2, and the reference acceleration is 32.174 ft/s^2 for the pound-mass. By agreement among the measurement standards laboratories of English-speaking countries, 1 lbm is also equivalent to 0.45359237 kg.

Despite the fact that the pound-mass and pound denote different physical quantities (mass and force), they are often improperly interchanged. One reason for the confusion is the similarity of their names. Another reason is related to the definition itself of the pound-mass: A quantity of matter having a mass of 1 lbm also weighs 1 lb, assuming Earth's gravity. By contrast, an object having a mass of 1 slug weighs 32.174 lb on Earth. You should realize, however, that the USCS is not alone with the dubious potential of confusing mass and weight. One will sometimes see the SI's kilogram used improperly to denote force. Some tires and pressure gages, for instance, are labeled with inflation pressures having the dimensions of kg/m^2. Some scales that are used in commerce have weights tabulated in kilograms or in terms of a defunct unit called the kilogram-force that is not even part of the SI.

Aside from mass, other derived units can be formed as combinations of the USCS's base units. Some that arise in mechanical engineering are listed in Table 3.5, which includes the *mil* (1000th of an inch, or 1/12,000th of a foot), the *foot-pound* (for energy, work, or heat), and the *horsepower*

Focus On MASS AND WEIGHT

Mass is an intrinsic property of an object based on the amount and density of material from which it is made. Mass m measures the quantity of matter that is contained in the object, and, as such, it does not vary with position, motion, or changes in the object's shape. Weight, on the other hand, is the force that is needed to support the object against gravitational attraction, and it is calculated as

$$w = mg$$

based on the gravitational acceleration

$$g = 32.174 \frac{\text{ft}}{\text{s}^2} \approx 32.2 \frac{\text{ft}}{\text{s}^2} \qquad \text{(USCS)}$$

$$g = 9.8067 \frac{\text{m}}{\text{s}^2} \approx 9.81 \frac{\text{m}}{\text{s}^2} \qquad \text{(SI)}$$

By international agreement, these accelerations are standard values at sea level and a latitude of 45°. The gravitational acceleration at a specific location on the Earth's surface, however, does vary with latitude, the slightly irregular shape of the Earth, the density of the Earth's crust, and the size of any nearby land masses. Although an object's weight depends on gravitational acceleration, its mass does not. For most mechanical engineering calculations, it is sufficient to approximate g to three significant digits.

Table 3.5
Certain Derived Units in the USCS

Quantity	Derived Unit	Abbreviation	Definition
Length	mil	mil	1 mil = 0.001 in.
	inch	in.	1 in. = 0.0833 ft
	mile	mi	1 mi = 5280 ft
Volume	gallon	gal	1 gal = 0.1337 ft^3
Mass	slug	slug	1 slug = 1 (lb ? s^2)/ft
	pound-mass	lbm	1 lbm = 3.1081 3 10^{-2} (lb ? s^2)/ft
Force	ounce	oz	1 oz = 0.0625 lb
	ton	ton	1 ton = 2000 lb
Torque, or moment of a force	foot-pound	ft · lb	—
Pressure or stress	pound/$inch^2$	psi	1 psi = 1 lb/in^2
Energy, work, or heat	foot-pound	ft · lb	—
	British thermal unit	Btu	1 Btu = 778.2 ft ? lb
Power	horsepower	hp	1 hp = 550 (ft ? lb)/s
Temperature	degree Fahrenheit	°F	°F = °R – 459.67

Although a change in temperature of 1° Rankine also equals a change of 1° F, numerical values are converted using the formula.

(550 (ft · lb)/s). Also note that the abbreviation for inch typically includes a period to distinguish it from the word "in" within technical documents.

Converting Between the SI and USCS

A numerical value in one unit system can be transformed into an equivalent value in the other system by using unit conversion factors. Conversion factors between some of the USCS and the SI quantities that arise in mechanical engineering are listed in Table 3.6 (see on page 92). The conversion process requires changes made both to the numerical value and to the dimensions associated with it. Regardless, the physical quantity remains unchanged, being made neither larger nor smaller, since the numerical value and units are transformed together. In general terms, the procedure for converting between the two systems is as follows:

1. Write the given quantity as a number followed by its dimensions, which could involve a fractional expression such as kg/s or N/m.
2. Identify the units desired in the final result.

Table 3.6

Conversion Factors between Certain Quantities in the USCS and SI

Quantity	Conversion		
Length	1 in.	=	25.4 mm
	1 in.	=	0.0254 m
	1 ft	=	0.3048 m
	1 mi	=	1.609 km
	1 mm	=	3.9370×10^{22} in.
	1 m	=	39.37 in.
	1 m	=	3.2808 ft
	1 km	=	0.6214 mi
Area	1 in^2	=	645.16 mm^2
	1 ft^2	=	9.2903×10^{22} m^2
	1 mm^2	=	1.5500×10^{23} in^2
	1 m^2	=	10.7639 ft^2
Volume	1 ft^3	=	2.832×10^{22} m^3
	1 ft^3	=	28.32 L
	1 gal	=	3.7854×10^{23} m^3
	1 gal	=	3.7854 L
	1 m^3	=	35.32 ft^3
	1 L	=	3.532×10^{22} ft^3
	1 m^3	=	264.2 gal
	1 L	=	0.2642 gal
Mass	1 slug	=	14.5939 kg
	1 lbm	=	0.45359 kg
	1 kg	=	6.8522×10^{22} slugs
	1 kg	=	2.2046 lbm
Force	1 lb	=	4.4482 N
	1 N	=	0.22481 lb
Pressure or stress	1 psi	=	6895 Pa
	1 psi	=	6.895 kPa
	1 Pa	=	1.450×10^{24} psi
	1 kPa	=	0.1450 psi

Table 3.6 *continued*

Quantity	Conversion		
Work, energy, or heat	1 ft · lb	=	1.356 J
	1 Btu	=	1055 J
	1 J	=	0.7376 ft · lb
	1 J	=	9.478×10^{24} Btu
Power	1 (ft · lb)/s	=	1.356 W
	1 hp	=	0.7457 kW
	1 W	=	0.7376 (ft · lb)/s
	1 kW	=	1.341 hp

3. If derived units such as L, Pa, N, lbm, or mi are present in the quantity, you may find it necessary to expand them in terms of their definitions and base units. In the case of the pascal, for instance, we would write

$$\text{Pa} = \frac{\text{N}}{\text{m}^2} = \left(\frac{1}{\text{m}^2}\right)\left(\frac{\text{kg ? m}}{\text{s}^2}\right) = \frac{\text{kg}}{\text{m ? s}^2}$$

where we canceled the meter dimension algebraically.

4. Likewise, if the given quantity includes a prefix that is not incorporated in the conversion factors, expand the quantity according to the prefix definitions listed in Table 3.3. The kilonewton, for instance, would be expanded as 1 kN = 1000 N.
5. Look up the appropriate conversion factor from Table 3.6, and multiply or divide, as necessary, the given quantity by it.
6. Apply the rules of algebra to cancel dimensions in the calculation and to reduce the units to the ones that you want in the final result.

You cannot escape conversion between the USCS and SI, and you will not find your way through the maze of mechanical engineering without being proficient with both sets of units. The decision as to whether the USCS or SI should be used when solving a problem will depend on how the information in the statement is specified. If the information is given in the USCS, then you should solve the problem and apply formulas using the USCS alone. Conversely, if the information is given in the SI, then formulas should be applied using the SI alone. It is bad practice for you to take data given in the USCS, convert to SI, perform calculations in SI, and then convert back to USCS (or vice versa). The reason for this recommendation is twofold. First, from the practical day-to-day matter of being a competent engineer, you will need to be fluent in both the USCS and SI. Furthermore, the additional steps involved when quantities are converted from one system to another and back again are just other opportunities for errors to creep into your solution.

Example 3.1 Engine Power Rating

A gasoline-powered engine produces a peak output of 10 hp. Express the given power P in the SI.

Approach

As listed in Table 3.5 for derived units in the USCS, the abbreviation "hp" refers to horsepower. The SI unit for power is the watt (W). The final row of Table 3.6 lists the conversions for power, and we read off the factor 1 hp = 0.7457 kW. Here the "kilo" prefix in Table 3.3 denotes a multiplier of 1000.

Solution

Applying the conversion factor to the engine's power rating, we have

$$\begin{aligned} P &= (10\ \text{hp})\left(0.7457\ \frac{\text{kW}}{\text{h}}\right) \\ &= 7.457(\cancel{\text{hp}})\left(\frac{\text{kW}}{\cancel{\text{h}}}\right) \\ &= 7.457\ \text{kW} \end{aligned}$$

Discussion

The conversion process involves two steps: algebraically combining the numerical values and the dimensions. In our calculation, we canceled the hp dimension, and we explicitly showed that step in our solution. In terms of the derived unit watt, the motor produces an output of 7457 W. However, since this numerical value is greater than 1000, the prefix "kilo" is used.

$$P = 7.457\ \text{kW}$$

Example 3.2 Fire Spinkler

The specification for a certain residential fire-suppression system is that water should be sprayed at the rate q of 10 gal/min. For the revision of a technical manual intended for customers outside the United States, express the flow rate in the SI based on a time interval of 1 s.

Approach

To complete this problem, we need to apply conversion factors for both volume and time. Referring to Table 3.6, we could express volume in the SI using the dimensions of either m^3 or L. We assume that a cubic meter is much larger than the amount of water we expect to be sprayed each second;

Example 3.2 *continued*

so we initially decide to convert volume to liters with the conversion factor 1 gal = 3.785 L. We will double-check that assumption after we complete the calculation.

Solution

Converting dimensions for both volume and time,

$$q = \left(10 \frac{\text{gal}}{\text{min}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right)\left(3.785 \frac{\text{L}}{\text{gal}}\right)$$

$$= 0.6308 \left(\frac{\text{gal}}{\text{min}}\right)\left(\frac{\text{min}}{\text{s}}\right)\left(\frac{\text{L}}{\text{gal}}\right)$$

$$= 0.6308 \frac{\text{L}}{\text{s}}$$

Discussion

We first combined the numerical values and then algebraically canceled the dimensions. It is good practice (and a good double-check) to show how the units clearly cancel in the conversion process. We could have alternatively expressed the flow rate in the units of m^3/s. However, since a cubic meter is 1000 times larger than a liter, the dimension L/s turns out to be better suited for the problem at hand since the numerical value of 0.6308 does not involve a power-of-ten exponent.

$$q = 0.6308 \frac{\text{L}}{\text{s}}$$

Helium-neon lasers are used in engineering laboratories, in robot vision systems, and even in the barcode readers found in supermarket checkout counters. A certain laser has a power output of 3 mW and produces light of wavelength λ = 632.8 nm. The lowercase Greek character lambda (λ) is a conventional symbol used for wavelength; Appendix A summarizes the names and symbols of other Greek letters. (a) Convert the power rating to horsepower. (b) Convert the wavelength to inches.

Approach

Referring to Table 3.3, the power dimension mW refers to a milliwatt or 10^{-3} W, and nm denotes a billionth of a meter (10^{-9} m). The conversion factors for power and length are listed in Table 3.6 as 1 kW = 1.341 hp and 1 m = 39.37 in.

Example 3.3 *continued*

Solution

(a) We first convert the power's SI prefix from milli to kilo in order to apply the conversion factor as listed in the table. The laser produces 3×10^{-3} W $= 3 \times 10^{-6}$ kW. We convert this small quantity into the USCS as follows:

$$P = (3 \times 10^{-6}\ \text{kW})\left(1.341\ \frac{\text{hp}}{\text{kW}}\right)$$

$$= 4.023 \times 10^{-6}\ (\cancel{\text{kW}})\left(\frac{\text{hp}}{\cancel{\text{kW}}}\right)$$

$$= 4.023 \times 10^{-6}\ \text{hp}$$

(b) Expressed in scientific notation, the laser's wavelength is 632.8×10^{-9} m $= 6.328 \times 10^{-7}$ m, and the length conversion becomes

$$\lambda = (6.328 \times 10^{-7}\ \text{m})\left(39.37\ \frac{\text{in.}}{\text{m}}\right)$$

$$= 2.491 \times 10^{-5}\ (\cancel{\text{m}})\left(\frac{\text{in.}}{\cancel{\text{m}}}\right)$$

$$= 2.491 \times 10^{-5}\ \text{in.}$$

Discussion

Because the dimensions of horsepower and inch are so much larger than the laser's power and wavelength, they are not very convenient for describing its characteristics.

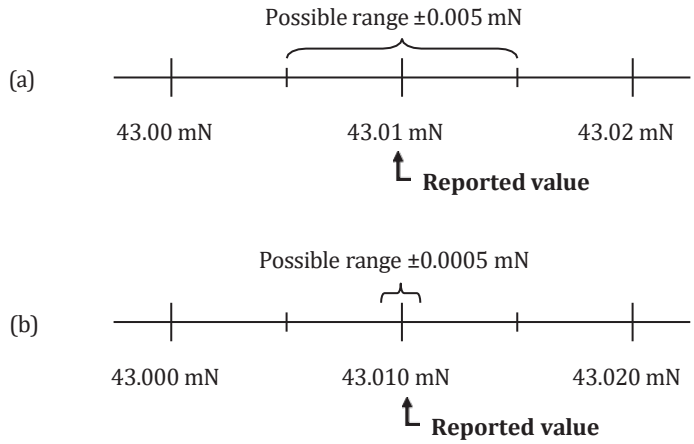
$$P = 4.023 \times 10^{-6}\ \text{hp}$$
$$\lambda = 2.491 \times 10^{-5}\ \text{in.}$$

3.4 Significant Digits

A *significant digit* is one that is known to be correct and reliable in the light of inaccuracy that is present in the supplied information, any approximations that have been made along the way, and the mechanics of the calculation itself. As a general rule, the last significant digit that you report in the answer to a problem should have the same order of magnitude as the last significant digit in the given data. It would be inappropriate to report more significant digits in the answer than were given in the supplied information, since that implies that the output of a calculation is somehow more accurate than the input to it.

Figure 3.4

The precision of a numerical value for force depends on the number of significant digits that are reported. The actual physical value lies in a range centered around the reported value. (a) Two trailing digits. (b) Three trailing digits.



Precision

The *precision* of a number is half as large as the place of the last significant digit present in the number. The factor is one-half because the last digit of a number represents the rounding off of the trailing digits, either higher or lower. For instance, suppose an engineer records in a design notebook that the force acting on the bearing of a hard disk drive's motor caused by rotational imbalance is 43.01 mN. That statement means that the force is closer to 43.01 mN than it is to either 43.00 mN or 43.02 mN. The reported value of 43.01 mN and its number of significant digits mean that the actual physical value of the force could lie anywhere between 43.005 and 43.015 mN [Figure 3.4(a)]. The precision of the numerical value is ±0.005 mN, the variation that could be present in the force reading and still result in a rounded-off value of 43.01 mN. Even when we write 43.00 mN, a numerical value that has two trailing zeros, four significant digits are present, and the implied precision remains ±0.005 mN.

Alternatively, suppose that the engineer had written the force as being 43.010 mN. That statement implies that the value is known quite accurately indeed, and means that the force lies closer to 43.010 mN than to either 43.009 mN or 43.011 mN. The precision is now ±0.0005 mN [Figure 3.4(b)].

Likewise, you can see how some ambiguity arises when a quantity such as 200 lb is reported. On the one hand, the value could mean that the measurement was made only within the nearest 100 pounds, meaning that the actual magnitude of the force is closer to 200 lb than to either 100 lb or 300 lb. On the other hand, the value could mean that the force is indeed 200 lb, and not 199 lb or 201 lb, and that the real value lies somewhere between 199.5 lb and 200.5 lb. Either way, it's vague to write a quantity such as 200 lb without giving some other indication as to the precision of the number.

As a general rule, during the intermediate steps of a calculation, retain several more significant digits than you expect to report in the final answer. In that manner, rounding errors will not creep into your solution, compound along the way, and distort the final answer. When the calculation is complete, you can

Chapter-2

Forces in Structures and Machines

Break a force down into its rectangular and polar components.

Determine the resultant of a system of forces by using the vector algebra and polygon methods.

Calculate the moment of a force using the perpendicular lever arm and moment component methods.

Understand the requirements for equilibrium, and be able to calculate unknown forces in simple structures and machines.

From the design standpoint, explain the circumstances in which one type of rolling element bearing would be selected for use over another, and calculate the forces acting on them.

4.1 OVERVIEW

Element 3: Forces in structures and machines

When mechanical engineers design products, systems, and hardware, they must apply mathematics and physical principles to model, analyze, and predict system behavior. Successful design is supported by effective engineering analysis; effective engineering analysis relies on an understanding of *forces in structures and machines*. This is the focus of this chapter and the next element of mechanical engineering.

This chapter introduces you to the subject of mechanics, a topic that encompasses forces that act on structures and machines and their tendency either to remain stationary or move. The fundamental principles that form the basis of mechanics are Newton's three laws of motion:

1. Every object remains in a state of rest or uniform motion of constant velocity unless an external unbalanced force acts upon it.
2. An object of mass m, subject to a force F, experiences an acceleration in the same direction as the force with a magnitude directly proportional to the magnitude of the force and inversely proportional to the mass of the object. This relationship can be expressed as $F = ma$.
3. The forces of action and reaction between two objectives are equal, opposite, and collinear.

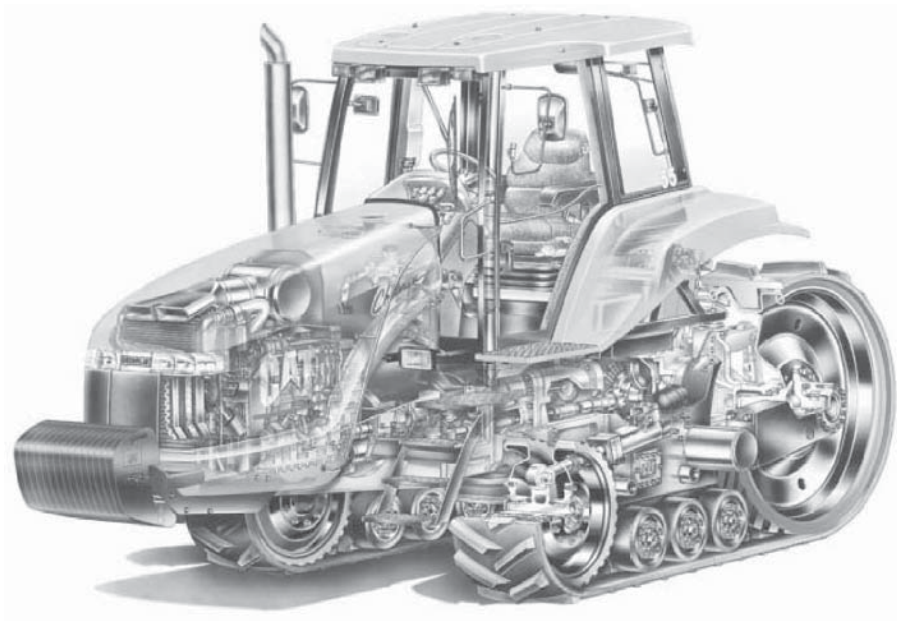


Figure 4.1

Heavy construction equipment is designed to support the large forces developed during operation.

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In this and the following chapters, we will explore these principles of forces and the problem-solving skills that are needed to understand their effects on engineering hardware. After developing the concepts of force systems, moments, and static equilibrium, you will see how to calculate the magnitudes and directions of forces acting on and within simple structures and machines. In short, the process of analyzing forces is a first step taken by engineers to see whether a certain piece of hardware will operate reliably (Figure 4.1).

A second objective of this chapter is for you to start understanding the inner workings of mechanical hardware, beginning with rolling element bearings. Just as an electrical engineer might select off-the-shelf resistors, capacitors, and transistors as the elements of a circuit, mechanical engineers have good intuition for specifying bearings, shafts, gears, belts, and other machine components. A working knowledge of hardware and machine components is important for you to develop a technical vocabulary. Mechanical engineering has its own precise language, and, to communicate effectively with other engineers, you will need to learn, adopt, and share that language. That background is also necessary to select the proper component: Should a ball, roller, tapered roller, or thrust roller bearing be used in this design?

The topics of force systems and machine components discussed in this chapter fit naturally into the hierarchy of mechanical engineering topics outlined in Figure 4.2. The topics fall under the Engineering sciences and analysis branch but provide support for key decisions in the design of

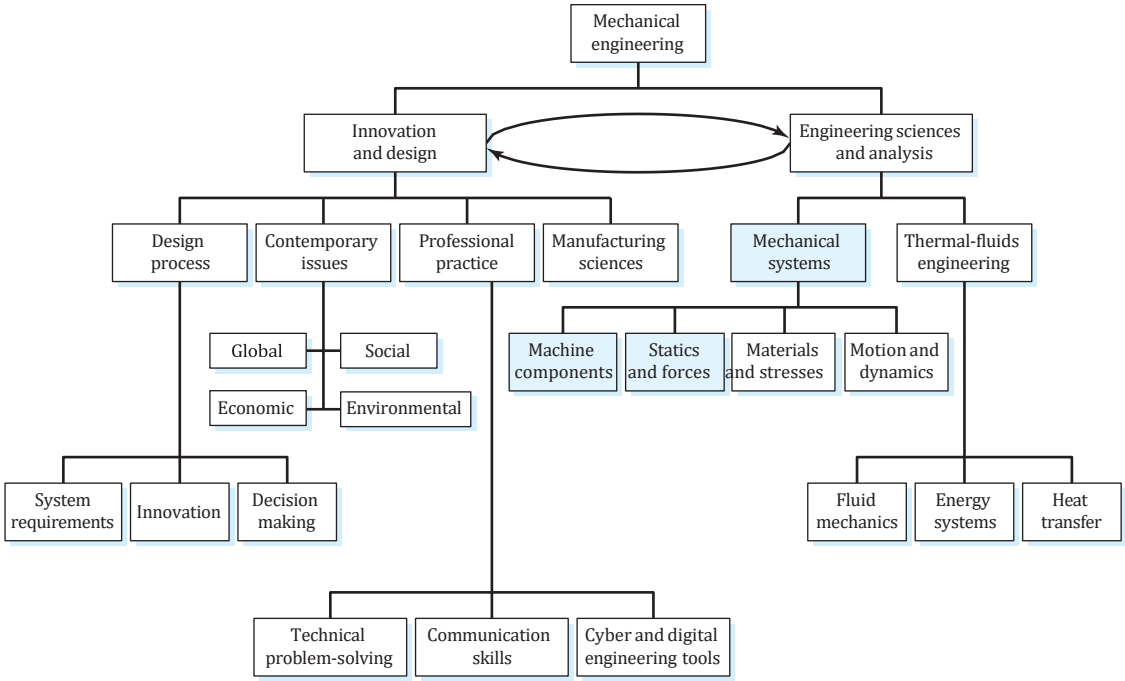


Figure 4.2 Relationship of the topics emphasized in this chapter (shaded boxes) relative to an overall program of study in mechanical engineering

innovative systems. Of course, in an introductory textbook, it is not possible to describe every machine and component that embodies mechanical engineering principles, and that is not our intention here or in later chapters. However, by examining just a few machine components, you will develop a growing appreciation for mechanical design issues. It's intellectually healthy for you to be curious about products, wonder how they were made, dissect them, and think about how they could have been made differently or better. In this chapter, we begin that journey by discussing various types of bearings and the forces that act on them. In Chapter 8, we continue that discussion with descriptions of gear, belt, and chain drives.

4.2 Forces in Rectangular and Polar Forms

Before we can determine the influence of forces on a structure or machine, we first need to describe a force's magnitude and direction. Our analysis will be limited to situations where the forces present all act in the same plane. The corresponding concepts and solution techniques for such two-dimensional problems carry over to the general case of structures and machines in three dimensions, but, for our purposes, it's better to avoid the added complexity

Table 4.1

Conversion Factors Between USCS and SI Units for Force

lb	oz	N
1	16	4.448
0.0625	1	0.2780
0.2248	3.597	1

in algebra and geometry. The properties of forces, equilibrium, and motion in three dimensions are also subjects that you will encounter later in the mechanical engineering curriculum.

Forces are vector quantities since their physical action involves both direction and magnitude. The magnitude of a force is measured by using the dimensions of pounds (lb) or ounces (oz) in the USCS and newtons (N) in the SI. In Chapter 3, the conversion factors between pounds and newtons were listed in Table 3.6, and they are shown in a slightly different format in Table 4.1. This style of listing the conversion factors is a compact way to depict the USCS-to-SI and SI-to-USCS conversion factors. Each row of the table contains equivalent quantities in the units shown at the top of the columns. The three rows of Table 4.1 mean the following:

Row 1: 1 lb 5 16 oz 5 4.448 N

Row 2: 0.0625 lb 5 1 oz 5 0.2780 N

Row 3: 0.2248 lb 5 3.597 oz 5 1 N

In this chapter and the following ones, we will use conversion tables having this type of format for other engineering quantities.

Rectangular Components

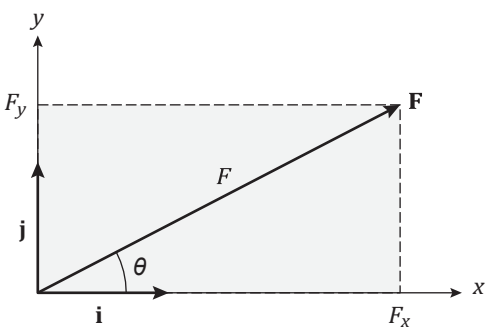
Vector notation

Force *vectors* are denoted by using boldface notation, as in **F**. One of the common methods used to represent the influence of a force is in terms of its horizontal and vertical components. Once we set the directions for the x- and y-axes, the force **F** can be broken down into its *rectangular components* along those directions. In Figure 4.3, the projection of **F** in the horizontal direction (the x-axis) is called F_x, and the vertical projection (y-axis) is called F_y. When

Rectangular components

Figure 4.3

Representing a force vector in terms of its rectangular components (F_x, F_y), and its polar components (F, θ).



you assign numerical values to F_x and F_y, you have described everything about the force **F**. In fact, the pair of numbers (F_x, F_y) is just the coordinates of the force vector's tip.

Unit vectors

The *unit vectors* **i** and **j** are used to indicate the directions in which F_x and F_y act. Vector **i** points along the positive x-direction, and **j** is a vector oriented in the positive y-direction. Just as F_x and F_y provide information about the magnitudes of the horizontal and vertical components, the unit vectors give information about the directions of those components. The unit vectors are so named because they have a length of one. By combining the rectangular components and unit vectors, the force is represented in vector algebra notation as

$$\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j} \tag{4.1}$$

Polar Components

In an alternative view, rather than thinking about a force in terms of how hard it pulls rightward and upward, you could tell someone how hard the force pulls, and in which direction it does so. This view is based on polar coordinates. As shown in Figure 4.3, **F** acts at the angle θ, measured relative to the horizontal axis. The length of the force vector is a scalar or simple numerical value, and it is denoted by $F = u\mathbf{F}u$. The u u notation designates the vector's magnitude F, which we write in plain typeface. Instead of specifying F_x and F_y, we can now view the force vector **F** in terms of the two numbers F and θ. This representation of a vector is called the *polar component* or magnitude-direction form.

Polar components

Magnitude
Direction

The force's *magnitude* and *direction* are related to its horizontal and vertical components through

$$F_x = F\cos\theta \qquad \text{(polar to rectangular)}$$
$$F_y = F\sin\theta \tag{4.2}$$

Trigonometric equations of this sort are reviewed in Appendix B. If we happen to know the force's magnitude and direction, these equations are used to determine its horizontal and vertical components. On the other hand, when we know F_x and F_y, the magnitude and direction of the force are calculated from

$$F = \sqrt{F_x^2 + F_y^2} \qquad \text{(rectangular to polar)}$$
$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) \tag{4.3}$$

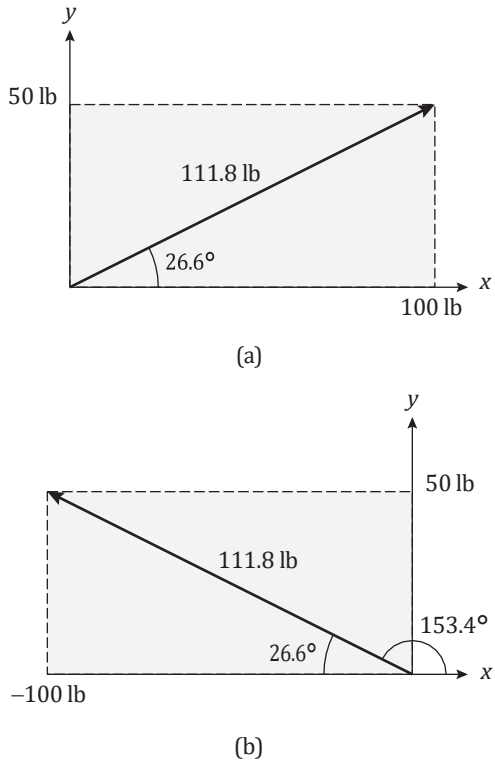
Principal value

The inverse tangent operation in Equation (4.3) calculates the *principal value* of its argument and returns an angle between 290° and 190°. Directly applying the equation $\theta = \tan^{-1}(F_y/F_x)$ will result in a value of θ that lies in either the first or fourth quadrants of the x-y plane.

Of course, a force could be oriented in any of the plane's four quadrants. In solving problems, you will need to examine the positive or

Figure 4.4

Determining the angle of action for a force that (a) lies in the first quadrant and (b) lies in the second quadrant.



negative signs of F_x and F_y and use them to determine the correct quadrant for θ. For instance, in Figure 4.4(a), where F_x 5 100 lb and F_y 5 50 lb, the force's angle of action is calculated as θ 5 $\tan^{21}(0.5)$ 5 26.6°. That numerical value correctly lies in the first quadrant because F_x and F_y—the coordinates of the force vector's tip—are both positive. On the other hand, in Figure 4.4(b), when F_x 5 2100 lb and F_y 5 50 lb, you might be tempted to report $\tan^{21}(20.5)$ 5 226.6°. That angle actually falls in the fourth quadrant, and it is incorrect as a measure of the force's orientation relative to the positive x-axis. As is evident from Figure 4.4(b), **F** forms an angle of 26.6° relative to the negative x-axis. The correct value for the force's angle of action relative to the positive x-axis is θ 5 180° 2 26.6° 5 153.4°.

4.3 Resultant of Several Forces

Force system

Resultant

A *force system* is a collection of several forces that simultaneously act on a structure or machine. Each force is combined with the others to describe their net effect, and the *resultant* **R** measures that cumulative action. As an example, consider the mounting post and bracket of Figure 4.5. The three forces $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ act in different directions and with different magnitudes. To determine whether the post is capable of supporting those forces, an

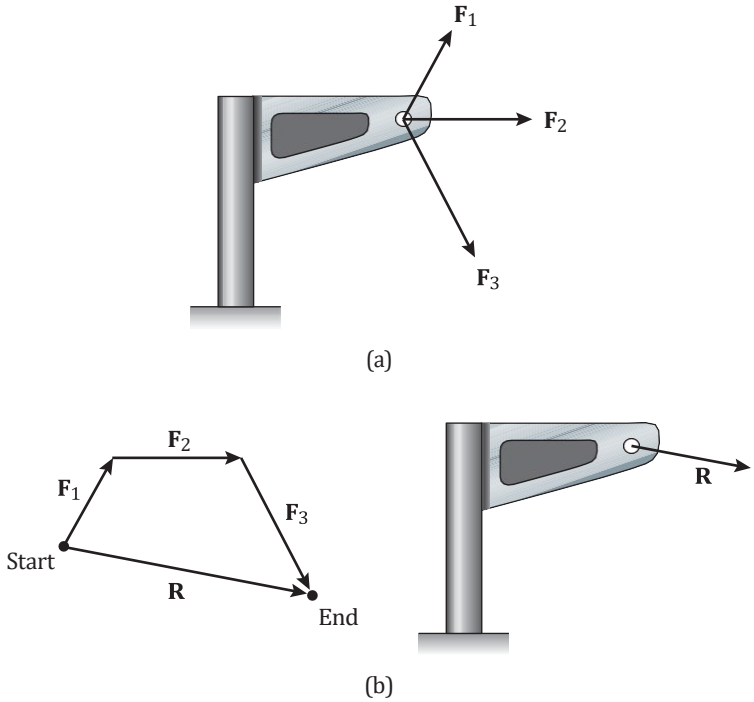


Figure 4.5

(a) A mounting post and bracket that are loaded by three forces. (b) The resultant **R** extends from the start to the end of the chain formed by adding $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ together.

engineer first needs to determine their net effect. With N individual forces $\mathbf{F}_i$ ($i = 1, 2, \ldots, N$) being present, they are summed according to

$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 + \cdots + \mathbf{F}_N = \sum_{i=1}^{N} \mathbf{F}_i \tag{4.4}$$

by using the rules of vector algebra. This summation can be carried out by the vector algebra or vector polygon approaches, which are described next. In different problem-solving situations, you will find that one approach might be simpler than another and that the two approaches can also be used for double-checking your calculations.

Vector Algebra Method

In this technique, each force is broken down into its horizontal and vertical components, which we label as $F_{x,i}$ and $F_{y,i}$ for the ith force. The resultant's horizontal portion R_x is the sum of the horizontal components from all of the individual forces that are present:

$$R_x = \sum_{i=1}^{N} F_{x,i} \tag{4.5}$$

Likewise, we separately sum the vertical components by using the equation

$$R_y = \sum_{i=1}^{N} F_{y,i} \tag{4.6}$$

The resultant force is then expressed in vector form as $\mathbf{R} = R_x\mathbf{i} + R_y\mathbf{j}$. Similar to Equation (4.3), we apply the expressions

$$R = \sqrt{R_x^2 + R_y^2}$$
$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) \qquad (4.7)$$

to calculate the resultant's magnitude R and direction θ. As before, the actual value for θ is found after considering the positive and negative signs of R_x and R_y, so that θ lies in the correct quadrant.

Vector Polygon Method

An alternative technique for finding the cumulative influence of several forces is the *vector polygon method*. The resultant of a force system can be found by sketching a polygon to represent the addition of the $\mathbf{F}_i$ vectors. The magnitude and direction of the resultant are determined by applying rules of trigonometry to the polygon's geometry. Referring to the mounting post of Figure 4.5(a), the vector polygon for those three forces is drawn by adding the individual $\mathbf{F}_i$'s in a chain according to the *head-to-tail rule*.

Head-to-tail rule

In Figure 4.5(b), the starting point is labeled on the drawing, the three forces are summed in turn, and the endpoint is labeled. The order in which the forces are drawn on the diagram does not matter insofar as the final result is concerned, but diagrams will appear visually different for various addition sequences. The endpoint is located at the tip of the last vector added to the chain. As indicated in Figure 4.5(b), the resultant **R** extends from the start of the chain to its end. The action of **R** on the bracket is entirely equivalent to the combined effect of the three forces acting together. Finally, the magnitude and direction of the resultant are determined by applying trigonometric identities to the polygon's shape. Some of the relevant equations for right and oblique triangles are reviewed in Appendix B.

Scale drawings

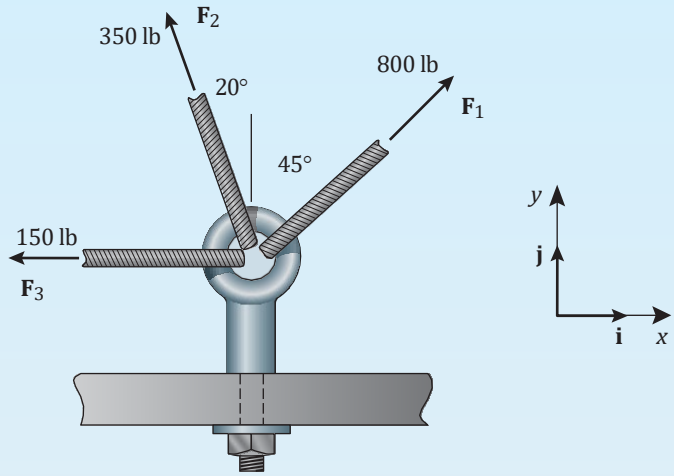
We can often obtain reasonably accurate results by summing vectors on a drawing that is made to *scale*, for instance, 1 in. on the drawing corresponds to 100 lb. Such drafting tools as a protractor, scale, and straightedge should be used to construct the polygon and to measure the magnitudes and directions of unknown quantities. It is certainly acceptable to use a purely graphical approach when solving engineering problems, provided that the drawing is large and precise enough that you can determine the answer to a fair number of significant digits.

Example 4.1 Cable Tie-Down

The eyebolt is fastened to a thick base plate, and it supports three steel cables with tensions 150 lb, 350 lb, and 800 lb. Determine the resultant force that acts on the eyebolt by using the vector algebra approach. The unit vectors **i** and **j** are oriented with the *x*-*y* coordinates as shown. (See Figure 4.6.)

Example 4.1 *continued*

Figure 4.6



Approach

We are tasked to find the resultant force on the eyebolt. By using Equations (4.1) and (4.2), we will break each force down into its horizontal and vertical components and write them in vector form. Then we will add the respective components of the three forces to find the resultant's components. Given those, the magnitude and angle of action of **R** follow from Equation (4.7).

Solution

The components of the 800-lb force are

$$F_{x,1} = (800 \text{ lb}) \cos 45° = 565.7 \text{ lb} \qquad \rightarrow [F_x = F \cos \theta]$$

$$F_{y,1} = (800 \text{ lb}) \sin 45° = 565.7 \text{ lb} \qquad \rightarrow [F_y = F \sin \theta]$$

and $\mathbf{F}_1$ is written in vector form as

$$\mathbf{F}_1 = 565.7\mathbf{i} + 565.7\mathbf{j} \text{ lb} \qquad \rightarrow [\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j}]$$

By using the same procedure for the other two forces,

$$\mathbf{F}_2 = -(350 \sin 20°)\mathbf{i} + (350 \cos 20°)\mathbf{j} \text{ lb}$$

$$= -119.7\mathbf{i} + 328.9\mathbf{j} \text{ lb}$$

$$\mathbf{F}_3 = -150\mathbf{i} \text{ lb}$$

Example 4.1 *continued*

To calculate the components of the resultant, the horizontal and vertical components of the three forces are summed separately:

$$R_x = 565.7 + 119.7 - 150 \text{ lb} \qquad \rightarrow \left[R_x = \sum_{i=1}^{N} F_{x,i}\right]$$

$$= 296.0 \text{ lb}$$

$$R_y = 565.7 + 328.9 \text{ lb} \qquad \rightarrow \left(R_y = \sum_{i=1}^{N} F_{y,i}\right)$$

$$= 894.6 \text{ lb}$$

The magnitude of the resultant force is

$$R = \sqrt{(296.0 \text{ lb})^2 + (894.6 \text{ lb})^2} \qquad \rightarrow \left[R = \sqrt{R_x^2 + R_y^2}\right]$$

$$= 942.3 \text{ lb}$$

and it acts at the angle

$$\theta = \tan^{-1}\left(\frac{894.6 \text{ lb}}{296.0 \text{ lb}}\right) \qquad \rightarrow \left[\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)\right]$$

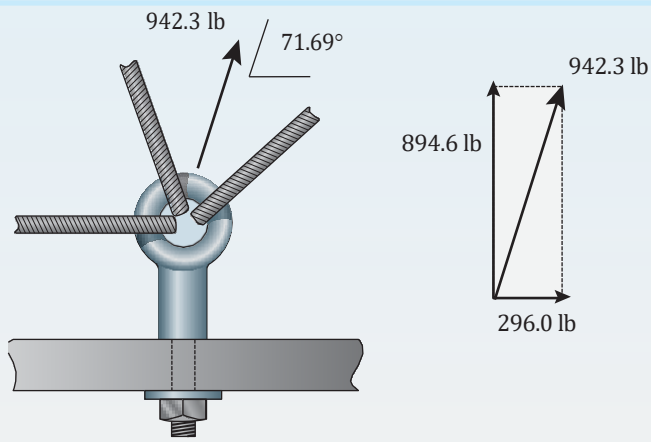
$$= \tan^{-1}\left(3.022 \frac{\text{lb}}{\text{lb}}\right)$$

$$= \tan^{-1}(3.022)$$

$$= 71.698$$

which is measured counterclockwise from the x-axis. (See Figure 4.7.)

Figure 4.7



Example 4.1 *continued*

Discussion

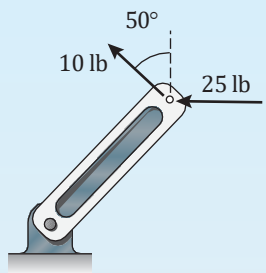
The resultant force is larger than any one force, but less than the sum of the three forces because a portion of $F_{x,3}$ cancels $F_{x,1}$. The resultant force acts upward and rightward on the bolt which is expected. The three forces acting together place the bolt in tension by pulling up on it, as well as bending it through the sideways loading R_x. As a double-check on dimensional consistency when calculating the resultant's angle, we note that the pound units in the argument of the inverse tangent function cancel. Since R_x and R_y are positive, the tip of the resultant vector lies in the first quadrant of the x-y plane.

$R = 942.3$ lb
$\theta = 71.69°$ counterclockwise from the x-axis

Example 4.2 Control Lever

The 10-lb and 25-lb forces are applied to the control lever of a mechanism. Determine the magnitude and direction of the resultant by using the vector polygon approach. (See Figure 4.8.)

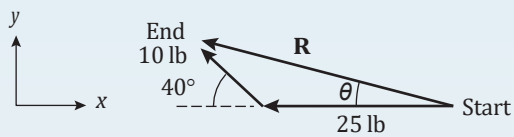
Figure 4.8



Approach

We are tasked to find the resultant force on the control lever. We first assume that the weight of the lever is negligible relative to the applied forces. We then sketch a vector polygon and combine the forces by using the head-to-tail rule. The two given forces, together with the resultant, will form a triangle. We can solve for the unknown length and angle in the triangle by applying the laws of cosines and sines from Appendix B. (See Figure 4.9.)

Figure 4.9



Example 4.2 *continued*

Solution

The 25-lb force is sketched first on the diagram, and the 10-lb force is added at the angle of 50° from vertical. The resultant **R** extends from the tail of the 25-lb force vector (which is labeled as the start point) to the head of the 10-lb force vector (the endpoint). The three vectors form a side-angle-side triangle. Applying the law of cosines, we solve for the unknown side length

$$R^2 = (10\text{ lb})^2 + (25\text{ lb})^2 \qquad \rightarrow [c^2 = a^2 + b^2 - 2ab\cos C]$$

$$-2(10\text{ lb})(25\text{ lb})\cos(180° - 40°)$$

from which $R = 33.29$ lb. The angle θ at which **R** acts is determined by applying the law of sines

$$\frac{\sin(180° - 40°)}{33.29\text{ lb}} = \frac{\sin\theta}{10\text{ lb}} \qquad \rightarrow \left[\frac{\sin A}{a} = \frac{\sin B}{b}\right]$$

The resultant acts at $\theta = 11.13°$ clockwise from the $2x$-axis.

Discussion

The resultant force is less than the sum of the two forces because the forces are not aligned in the same direction. Also, the direction of the resultant force, at an angle between the two forces, matches the expectation.To double-check the solution, we could break the two forces down into their horizontal and vertical components, as in the technique in Example 4.1. By using the x-y coordinate system, the vector expression for the 10-lb force becomes $\mathbf{F} = 27.660\mathbf{i} + 6.428\mathbf{j}$ lb. See if you can complete the double-check for R and θ by using the vector algebra method.

$R = 33.29$ lb
$\theta = 11.13°$ clockwise from the $2x$-axis

4.4 Moment of a Force

When you are trying to loosen a bolt, it is more easily turned if you use a wrench with a long handle. In fact, the longer the handle, the less force you need to apply with your hand. The tendency of a force to make an object rotate is called a *moment.* The magnitude of a moment depends both on the force applied and on the lever arm that separates the force from the point of rotation.

Perpendicular Lever Arm Method

The magnitude of a moment is defined by

$$M_o = Fd \tag{4.8}$$

Table 4.2 Conversion Factors Between USCS and SI Units for Moment or Torque

in · lb	ft · lb	N · m
1	0.0833	0.1130
12	1	1.356
8.851	0.7376	1

Perpendicular lever arm

where M_o is the moment of the force about point O and F is the magnitude of the perpendicular force. The distance d is called the *perpendicular lever arm*, and it extends from the force's line of action to point O. The term "torque" is sometimes used interchangeably to describe the effect of a force acting across a lever arm. However, mechanical engineers generally reserve *torque* to describe moments that cause rotation of a shaft in a motor, engine, or gearbox. We will discuss those applications in Chapter 8.

Torque

Based on Equation (4.8), the dimensions for a moment are the product of force and length. In the USCS, the unit for a moment is either in · lb or ft · lb. In the SI, the unit N · m is used, and various prefixes can be applied when the numerical value is either very large or very small. For instance, 5000 N · m 5 5 kN · m and 0.002 N · m 5 2 mN · m. Note that the dot symbol (·) is used to show multiplication between the dimensions; so it is clear that mN · m means millinewton-meters. Conversion factors between the two systems of units are listed in Table 4.2. Each row of the table is equivalent, and the first row indicates that

$$1 \text{ in} \cdot \text{lb} = 0.0833 \text{ ft} \cdot \text{lb} = 0.1130 \text{ N} \cdot \text{m}$$

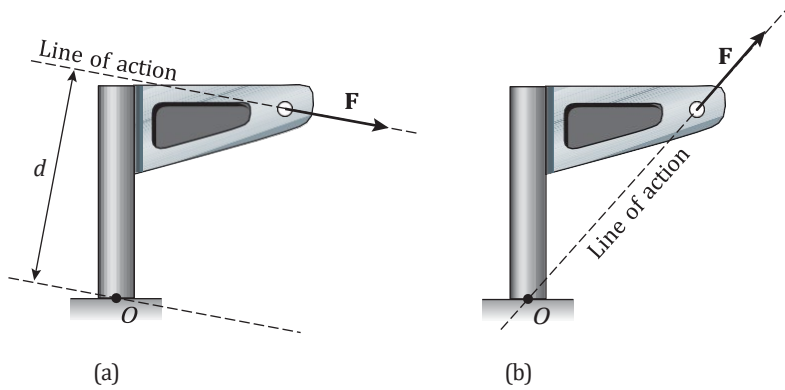
Work and energy are other quantities that arise in mechanical engineering, and they also have dimensions that are the product of force and length. When working in the SI, a joule (J) is defined as one newton-meter, and it is the amount of work that is performed by a 1-N force that acts through a distance of 1 m. However, the physical quantities of work and energy are quite different from moments and torques, and, to be clear about distinguishing between them, only the dimension N · m should be used in the SI for moment and torque.

The expression $M_o = Fd$ can be best understood by applying it to a specific structure. In Figure 4.10(a) (see on page 142), the force **F** is directed generally downward and to the right on the bracket. One might be interested in the moment of **F** about the base of the support post, which is labeled in the figure as point O. The post could break at that location, and an engineer would design the post's diameter and length to make sure that it can support **F**. The moment is calculated based on both the magnitude of **F** and the perpendicular offset d between the force's line of action and point O. The continuous straight line on which a force vector lies is called its *line of action*. In fact, **F** could be applied to the bracket at any point along its line of action, and the moment about O would be unchanged. The direction of the moment in Figure 4.10(a) is clockwise because **F** tends to cause the post to rotate that way (even though the rigid mounting would prevent the post from actually moving in this case).

Line of action

Figure 4.10

Calculating the moment of a force **F**. (a) The line of action of **F** is separated from O by the perpendicular lever arm distance d. (b) The line of action of **F** passes through O, and $M_o = 0$.



In Figure 4.10(b), the direction of the force has been changed. The force's line of action now passes exactly through point O, and so the offset distance is $d = 0$. No moment is produced, and the force tends to pull the post directly out of its base without rotating it. In short, the orientation of a force as well as its magnitude must be taken into account when you calculate a moment.

Moment Components Method

Just as we can break a force down into its rectangular components, it is sometimes useful to calculate a moment in terms of its components. The moment is determined as the sum of portions associated with the two components of the force, rather than the full resultant value of the force. One motivation for calculating the moment in this manner is that the lever arms for individual components are often easier to find than those for the entire resultant force. When applying this technique, we need to use a sign convention and keep track of whether the contribution made by each moment component is clockwise or counterclockwise.

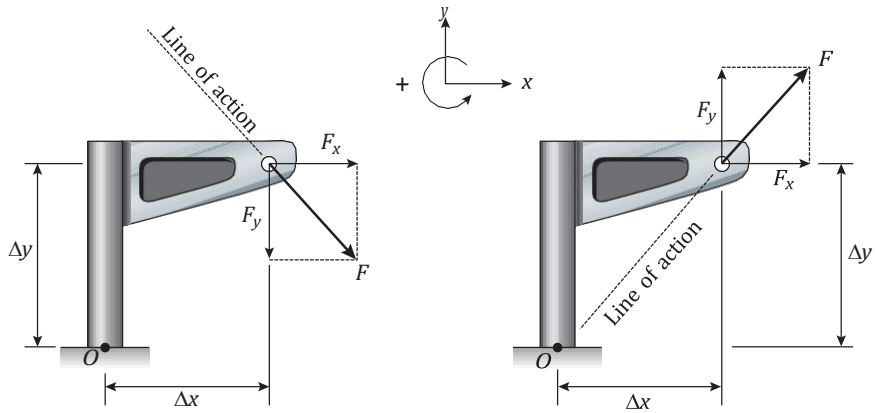
Moment sign convention

To illustrate this method, let's return to the example of the post and bracket with a force acting on it (Figure 4.11). We will choose the following *sign convention* for rotation directions: A moment that tends to cause counterclockwise rotation is positive, and a clockwise moment is negative. This choice of positive and negative directions is arbitrary; we could just as easily have selected the clockwise direction to be positive. However, once the sign convention is chosen, we need to stick with it and apply it consistently.

The force's components F_x and F_y are shown in the first case [Figure 4.11(a)]. Rather than determine the distance from point O to the line of action of **F**, which might involve a geometrical construction that we want to avoid, we instead calculate the individual lever arm distances for F_x and F_y, which are more straightforward. Keeping track of the sign convention, the moment about O becomes $M_o = -F_x \Delta y - F_y \Delta x$. The individual contributions to M_o are each negative because both F_x and F_y tend to cause clockwise rotation. Their effects combine constructively to produce the net moment.

Figure 4.11

(a) Both F_x and F_y create clockwise moments about point O. (b) F_x exerts a clockwise moment, but F_y exerts a counterclockwise moment.



The orientation of **F** has been changed in Figure 4.11(b). While F_x continues to exert a negative moment, F_y now tends to cause counterclockwise, or positive, rotation about O. The net moment becomes $M_o = -F_x\,\Delta y + F_y\,\Delta x$. Here the two components combine in a destructive manner. In fact, for the special orientation in which $\Delta y/\Delta x = F_y/F_x$, the two terms precisely cancel. The moment in that situation is zero because the line of action for **F** passes through O.

In using the moment components method, we generally write

$$M_o = \pm F_x\,\Delta y \pm F_y\,\Delta x \qquad (4.9)$$

where we imply that the numerical values for F_x, Δx, F_y, and Δy are all positive. The positive and negative signs in the equation are assigned while solving a problem depending on whether the moment component tends to cause clockwise or counterclockwise rotation.

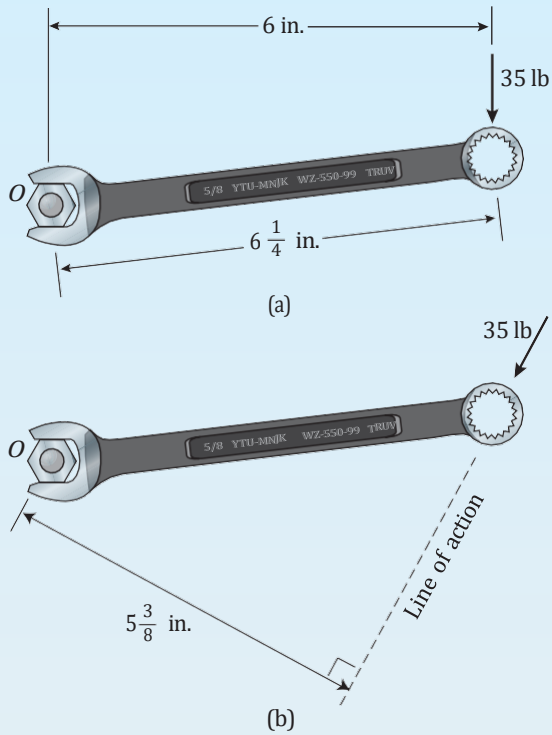
Regardless of whether you use the perpendicular lever arm method or the moment components method, when reporting an answer, you should be sure to state (1) the numerical value for the moment's magnitude, (2) the dimensions, and (3) whether the direction is clockwise or counterclockwise. You can indicate the direction by using a plus or minus (±), provided that you have defined the sign convention on your drawing.

Example 4.3 Open-Ended Wrench

The machinist's wrench is being used to tighten a hexagonal nut. Calculate the moments produced by the 35-lb force about the center of the nut when the force is applied to the wrench in the orientations (a) and (b) as shown. The overall length of the handle, which is inclined slightly upward, is $6\frac{1}{4}$ in. long between centers of the open and closed ends. Report your answer in the dimensions of ft · lb. (See Figure 4.12 on page 144.)

Example 4.3 *continued*

Figure 4.12



Approach

We are tasked with calculating the moments in two orientations. We first assume that we can neglect the moment created by the weight of the wrench. Then we will need to use the perpendicular distances in both cases. When the force acts straight down in case (a), the perpendicular distance from the center of the nut to the force's line of action is $d = 6$ in. The inclination and length of the wrench's handle are immaterial insofar as calculating d is concerned because the handle's length is not necessarily the same as the perpendicular lever arm's.

Solution

(a) The moment has magnitude

$$M_o = (35 \text{ lb})(6 \text{ in.}) \qquad \rightarrow [M_o = Fd]$$

$$= 210 \text{ in} \cdot \text{lb}$$

and it is directed clockwise. We apply the conversion factor between in ? lb and ft ? lb from Table 4.2 to convert M_o to the desired dimensions:

Example 4.3 *continued*

$$M_o = (210 \text{ in} \cdot \text{lb})\left(0.0833 \frac{\text{ft} \cdot \text{lb}}{\text{in} \cdot \text{lb}}\right)$$

$$= 17.50 \ (\cancel{\text{in} \cdot \text{lb}})\left(\frac{\text{ft} \cdot \text{lb}}{\cancel{\text{in} \cdot \text{lb}}}\right)$$

$$= 17.50 \text{ ft} \cdot \text{lb}$$

(b) In the second case, the force has shifted to an inclined angle, and its line of action has changed so that $d = 5\frac{3}{8}$ in. The moment is reduced to

$$M_o = (35 \text{ lb})(5.375 \text{ in.}) \qquad \rightarrow [M_o = Fd]$$

$$= 188.1 \text{ in} \cdot \text{lb}$$

Converting to the dimensions of ft · lb,

$$M_o = (188.1 \text{ in} \cdot \text{lb})\left(0.0833 \frac{\text{ft} \cdot \text{lb}}{\text{in} \cdot \text{lb}}\right)$$

$$= 15.67 \ (\cancel{\text{in} \cdot \text{lb}})\left(\frac{\text{ft} \cdot \text{lb}}{\cancel{\text{in} \cdot \text{lb}}}\right)$$

$$= 15.67 \text{ ft} \cdot \text{lb}$$

Discussion

In each case, the moments are clockwise, but the moment is smaller in case (b) because the perpendicular distance is less than in case (a). If gravity was accounted for, the clockwise moment in each case would be larger since gravity would create an additional force acting down around the middle of the wrench.

When we report the final answer, we indicate the numerical value, dimensions, and direction.

Case (a): $M_o = 17.50$ ft · lb (clockwise)

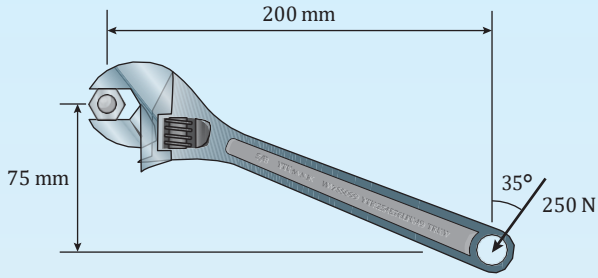
Case (b): $M_o = 15.67$ ft · lb (clockwise)

Example 4.4 Adjustable Wrench

Determine the moment about the center of the nut when the 250-N force is applied to the adjustable wrench. Use (a) the perpendicular lever arm method and (b) the moment components method. (See Figure 4.13 on page 146.)

Example 4.4 *continued*

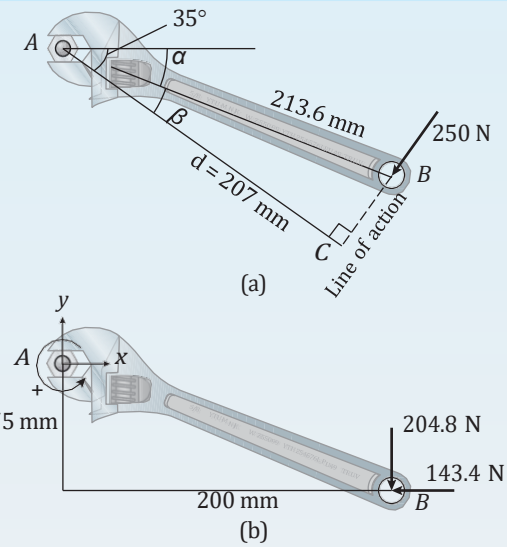
Figure 4.13



Approach

We are tasked with finding the resulting moment using two methods. We again assume that we can neglect the impact of the wrench's weight. We denote the center of the nut as point *A*, and the point of application of the force as point *B*. The moment is calculated by applying Equations (4.8) and (4.9). We will use the trigonometric equations from Appendix B to determine the necessary lengths and angles. (See Figure 4.14.)

Figure 4.14



Solution

(a) We first need to determine the length of the perpendicular lever arm, and this step involves geometrical constructions. By using the given dimensions,

$$AB = \sqrt{(75\ \text{mm})^2 + (200\ \text{mm})^2} \qquad \rightarrow z^2 = x^2 + y^2$$

$$= 213.6\ \text{mm}$$

Example 4.4 *continued*

Although this is the distance to the location at which the force is applied, it is not the perpendicular lever arm's length. For that, we need to calculate the length of *AC,* which is perpendicular to the force's line of action. Because the force is inclined by 35° from vertical, a line perpendicular to it is oriented 35° from horizontal. Line *AB* lies at the angle

$$\alpha = \tan^{-1}\left(\frac{75\ \text{mm}}{200\ \text{mm}}\right) \qquad \rightarrow \left[\tan\theta = \frac{y}{x}\right]$$
$$= \tan^{-1}\left(0.375\ \frac{\text{mm}}{\text{mm}}\right)$$
$$= \tan^{-1}(0.375)$$
$$= 20.568$$

below horizontal, and so it is offset by

$$\beta = 35° - 20.56° = 14.44°$$

from line *AC*. The perpendicular lever arm distance becomes

$$d = (213.6\ \text{mm})\cos 14.44° \qquad \rightarrow [x = z\cos\theta]$$
$$= 206.8\ \text{mm}$$

and the wrench's moment becomes

$$M_A = (250\ \text{N})(0.2068\ \text{m}) \qquad \rightarrow [M_A = Fd]$$
$$= 51.71\ \text{N}\cdot\text{m}$$

which is directed clockwise.

(b) In the moment components method, the 250-N force is broken down into its horizontal and vertical components having magnitudes (250 N) sin 35° = 143.4 N and (250 N) cos 35° = 204.8 N. Those components are oriented leftward and downward in the diagram. Individually, each exerts a clockwise moment about point *A*. Referring to our sign convention in the figure, a counterclockwise moment is positive. By summing the moment produced by each force component, we have

$$M_A = -(143.4\ \text{N})(0.075\ \text{m}) - (204.8\ \text{N})(0.2\ \text{m}) \qquad \rightarrow [M_A = -F_x\,\Delta y - F_y\,\Delta x]$$
$$= -251.71\ \text{N}\cdot\text{m}$$

Because the numerical value is negative, the net moment is directed clockwise.

Discussion

In this instance, it's probably easier to apply the moment components method because the horizontal and vertical dimensions of the wrench are

Example 4.4 *continued*

given in the problem's statement. Also, if gravity was accounted for, the clockwise moment would be larger because gravity would create an additional force acting down around the middle of the wrench.

$$M_A = 51.71 \text{ N} \cdot \text{m (clockwise)}$$

4.5 Equilibrium of Forces and Moments

With groundwork for the properties of forces and moments now in place, we next turn to the task of calculating (unknown) forces that act on structures and machines in response to other (known) forces that are present. This process involves applying the principles of static equilibrium from Newton's first law to structures and machines that are either stationary or moving at constant velocity. In either case, no acceleration is present, and the resultant force is zero.

Particles and Rigid Bodies

A mechanical system can include either a single object (for instance, an engine's piston) or multiple objects that are connected (the entire engine). When the physical dimensions are unimportant with respect to calculating forces, the object is called a *particle*. This concept idealizes the system as being concentrated at a single point rather than being distributed over an extended area or volume. For the purposes of solving problems, a particle can be treated as having negligible dimensions.

Particle

On the other hand, if the length, width, and breadth of an object are important for the problem at hand, it is called a *rigid body*. As an example, when looking at the motion of a communications satellite as it orbits the Earth, the spacecraft can be regarded as a particle because its dimensions are small compared to the size of the orbit. However, when the satellite is being launched and engineers are interested in the aerodynamics and flight characteristics of the rocket, the launch vehicle would be modeled as a rigid body. Figure 4.15 illustrates the conceptual distinction between forces applied to a particle and to a rigid body. You can see how a force imbalance could cause the rigid body, but not the particle, to rotate.

Rigid body

A particle is in equilibrium if the forces acting on it balance with zero resultant. Because forces combine as vectors, the resultant of the N forces that are present must be zero in two perpendicular directions, which we label x and y:

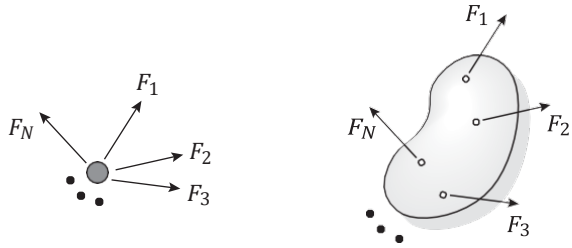
Force balance

$$\sum_{i=1}^{N} F_{x,i} = 0$$

$$\sum_{i=1}^{N} F_{y,i} = 0 \qquad (4.10)$$

Figure 4.15

A schematic of N forces acting on (a) a single particle and (b) a rigid body.



Moment balance

For a rigid body to be in equilibrium, the net moment must also be zero. When those conditions are met, there is no tendency for the object either to move in any direction in response to the forces or to rotate in response to the moments. The requirements for equilibrium of a rigid body comprise Equation (4.10) and

$$\sum_{i=1}^{N} M_{o,i} = 0 \tag{4.11}$$

The notation $M_{o,i}$ is used to denote the moment of the ith force that is present.

Sign conventions are a good bookkeeping method to distinguish forces that act in opposing directions and moments that are oriented clockwise or counterclockwise. The summations in the equilibrium equations extend over all forces and moments that are present, whether or not their directions and magnitudes are known in advance. Forces that are unknown at the start of the analysis are always included in the summation, algebraic variables are assigned to them, and the equilibrium equations are applied to determine numerical values.

Mathematically speaking, the equilibrium equations for a rigid body comprise a system of three linear equations that involve the unknown forces. One implication of this characteristic is that it is possible to determine at most three unknown quantities when Equations (4.10) and (4.11) are applied to a single rigid body. By contrast, when applying the equilibrium requirements to a particle, the moment equation is not used. In that case, only two *independent equations* exist, and two unknowns at most can be determined. It is not possible to obtain more independent equations of equilibrium by resolving moments about an alternative point or by summing forces in different directions. The additional equations still will be valid, but they simply will be combinations of the other, already derived, ones. As such, they will provide no new information. When you solve equilibrium problems, always check to be sure that you do not have more unknown quantities than available independent equations.

Independent equations

Free Body Diagrams

Free body diagrams are sketches used to analyze the forces and moments that act on structures and machines, and drawing them is an important skill. The free body diagram identifies the mechanical system being examined, and it represents all of the known and unknown forces present. Follow three main steps when drawing a free body diagram:

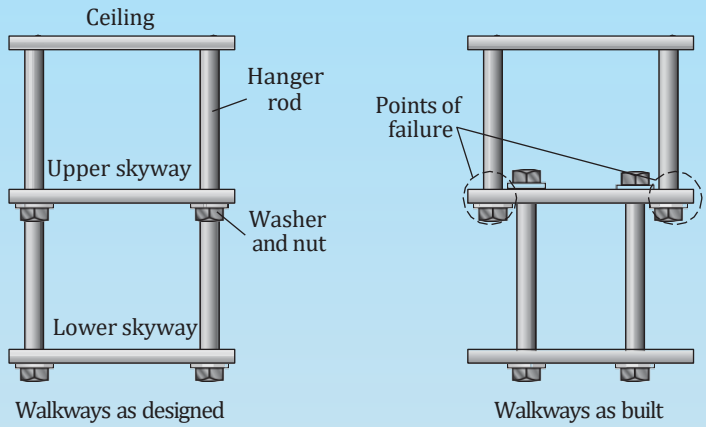
1. Select an object that will be analyzed by using the equilibrium equations. Imagine that a dotted line is drawn around the object, and note how the line would cut through and expose various forces. Everything within the dotted line is isolated from the surroundings and should appear on the diagram.
2. Next, the coordinate system is drawn to indicate the positive sign conventions for forces and moments. It is meaningless to report an answer of, say, 225 N · m or 1250 lb without having defined the directions associated with the positive and negative signs.
3. In the final step, all forces and moments are drawn and labeled. These forces might represent weight or contact between the free body and other objects that were removed when the body was isolated. When a force is known, its direction and magnitude should be written on the diagram. At this step in the analysis, forces are included even if their magnitudes and directions are not known. If the direction of a force is unknown (for instance, upward/downward or leftward/rightward), you should just draw it one way or the other on the free body diagram, perhaps using your intuition as a guide. After applying the equilibrium equations and consistently using a sign convention, the correct direction will be determined from your calculations. If you find the quantity to be positive, then you know that the correct direction was chosen at the outset. On the other hand, if the numerical value turns out to be negative, the result simply means that the force acts counter to the assumed direction.

Focus On Engineering Failure Analysis

We have seen how mechanical engineering is based on designing new products, but sometimes it also involves analyzing product failures. More broadly known as forensic engineering, this field was central to identifying the cause behind a catastrophic structural collapse that occurred in the summer of 1981.

At that time, the Hyatt Regency Hotel in Kansas City, Missouri, was a one-year-old facility that included a 40-story tower and a spacious, four-story, open-air atrium. Suspended from the ceiling and hanging above the main lobby area, three floating walkways (called skyways) enabled guests on the hotel's first few floors to view

Figure 4.16



and enjoy the expansive lobby from above. The second-floor's walkway was directly underneath the fourth-floor's walkway. One Friday evening, some 1500 to 2000 people were attending a party being held in the hotel's atrium. The majority of the partygoers were at the lobby's ground level, and many others were dancing above on the skyways. At 7:05 p.m., a loud "crack" was heard, and the connections that supported the fourth-floor's walkway suddenly broke. The upper (fourth-floor) walkway buckled into three sections, and collapsed onto the lower (second-floor) walkway. The entire structure, comprising some 100,000 lb of debris, fell to the crowded atrium below. In what is regarded as one of the worst structural failures in the United States, 114 people died and 185 were injured, many seriously.

What went wrong? After an extensive investigation, forensic engineers found that the skyway's design was changed without sufficient review once construction had already begun. (See Figure 4.16.) The original design called for the upper and lower skyways to be suspended from the ceiling by the same, continuous, $1\frac{1}{4}$ in.-diameter steel hanger rod. In turn, the walkways were to be supported by washers and nuts that were threaded onto the hanger rods. During construction, however, the single, long hanger rod was replaced by a pair of shorter rods that were offset from one another where they connected to the upper skyway. At first glance, the rationale behind the design change made sense; shorter rods were easier to acquire, and they could be installed with less risk of bending them. In place of a single hanger rod, the arrangement that was actually constructed consisted of two hanger rods: One extended from the ceiling to the upper floor, and the other connected the upper and lower floors. When the skyways collapsed, the washer, nut, and rod ripped through the upper walkway's structure at the outboard bolted connection. The hanger rods themselves did not break, and the upper rods remained attached to the ceiling.

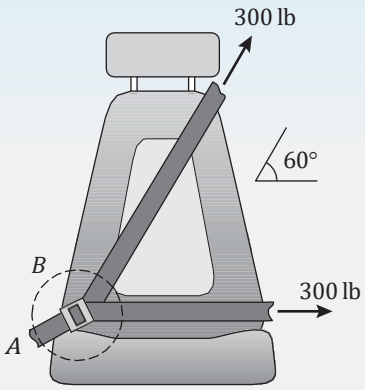
As it turns out, the seemingly innocuous design change doubled the force that had to be carried by the nut-and-washer connection between the upper hanger rod and the upper skyway. As described in Problems P4.24 and P4.25 at the end of this chapter, a back-of-the-envelope engineering calculation using free body diagrams shows that the force would, in fact, be doubled by changing the design from one hanger rod to two. Unfortunately, that force–balance calculation was evidently not performed by the responsible engineers.

What lessons can we take away from this case study? Changes to a design will sometimes occur after construction or manufacturing begins. Engineers have ethical responsibilities to be proactive by using engineering analysis to evaluate the design changes and to ensure that the changes do not compromise safety. An equilibrium force analysis using the principles described in this chapter would have exposed the flaw in the design change. The public trusts engineers and counts on them to do their jobs well, because the products and structures that engineers design affect people for better and, in this instance, for worse.

Example 4.5 Seat Belt Buckle

During crash testing of an automobile, the lap and shoulder seat belts develop tensions of 300 lb. Treating the buckle B as a particle, determine the tension T in the anchor strap AB, and the angle at which it acts. (See Figure 4.17.)

Figure 4.17

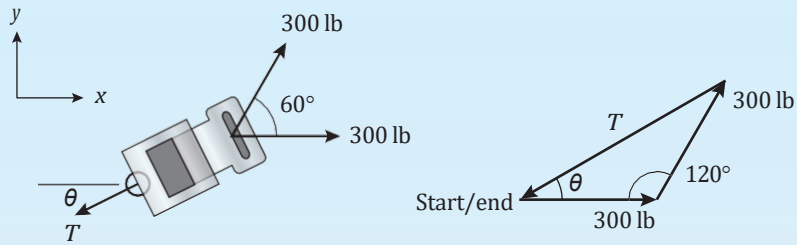


Approach

We are tasked with finding the magnitude and direction of the tension in the seat belt anchor strap. By treating the buckle as a particle, we can assume that all the force from the shoulder and lap belts act on the anchor strap. The weights of the straps are assumed to be negligible. The free body diagram of the buckle is drawn along with the x-y coordinate system to indicate our sign convention for the positive horizontal and vertical directions. Three forces act on the buckle: the two given 300-lb forces and the unknown force in the anchor strap. For the buckle to be in equilibrium, these three forces must balance. Although both the magnitude T and direction θ of the force in strap AB are unknown, both quantities are shown on the free body diagram for completeness. There are two unknowns, T and θ, and the two expressions in Equation (4.10) are available to solve the problem. (See Figure 4.18.)

Example 4.5 *continued*

Figure 4.18



Solution

We will combine the three forces by using the vector polygon approach. The polygon's start and endpoints are the same because the three forces combine to have zero resultant; that is, the distance between the polygon's start and endpoints is zero. The tension is determined by applying the law of cosines to the side-angle-side triangle (equations for oblique triangles are reviewed in Appendix B):

$$T^2 = (300\text{ lb})^2 + (300\text{ lb})^2 \qquad \rightarrow [c^2 = a^2 + b^2 - 2ab\cos C]$$

$$-2(300\text{ lb})(300\text{ lb})\cos 120°$$

We calculate $T = 519.6$ lb. The anchor strap's angle is found from the law of sines:

$$\frac{\sin\theta}{300\text{ lb}} = \frac{\sin 120°}{519.6\text{ lb}} \qquad \rightarrow \left[\frac{\sin A}{a} = \frac{\sin B}{b}\right]$$

and $\theta = 30°$.

Discussion

The three forces in the vector polygon form an isosceles triangle. As a double-check, the interior angles sum to 180°, as they should. In reality, the forces in the shoulder and lap belts will act on the buckle, which then acts on the anchor strap. By assuming the buckle to be a particle, we were able to simplify the analysis. In an alternative solution, we could break down the two 300-lb forces into their components along the x- and y-axes and apply Equation (4.10).

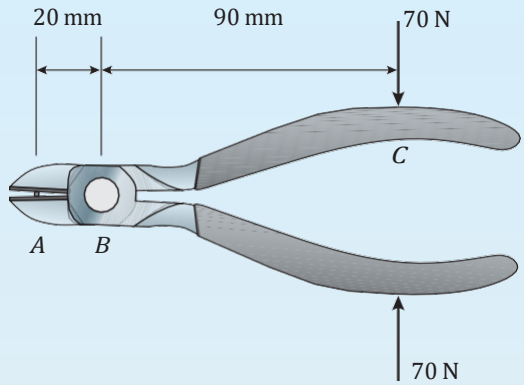
$T = 519.6$ lb

$\theta = 30°$ counterclockwise from the $-x$-axis

Example 4.6 Wire Cutters

A technician applies the 70-N gripping force to the handles of the wire cutters. What are the magnitudes of the cutting force on the electrical wire at *A* and the force carried by the hinge pin at *B*? (See Figure 4.19.)

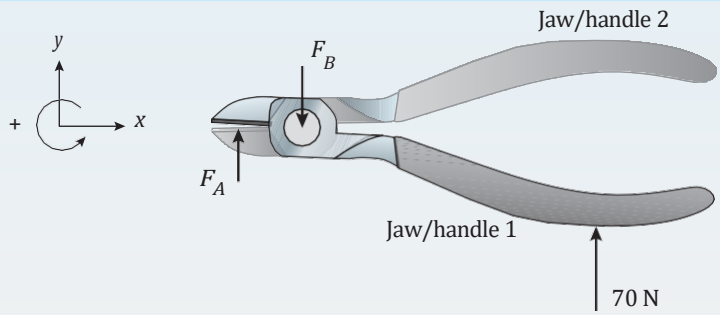
Figure 4.19



Approach

We are tasked to find the forces at the cutting point and the hinge. We assume that the weight of the wire cutters is negligible compared to the applied force. We then show the coordinate system and positive sign convention for the directions of forces and moments.The free body diagram is drawn for one jaw/handle assembly, which is a rigid body because it can rotate and the distances between the forces are significant to the problem. When a blade presses against the wire, the wire in turn pushes back on the blade following the principle of action–reaction. We label the jaw's cutting force as F_A, and the force exerted by the hinge pin on the jaw/handle as F_B. The 70-N gripping force is given, and it is also included on the free body diagram. (See Figure 4.20.)

Figure 4.20



Example 4.6 *continued*

Solution

The cutting force is found by applying the equilibrium equations for a rigid body. The force balance requirement in the vertical direction becomes

$$F_A + F_B - (70\ \text{N}) = 0 \qquad \rightarrow \left\{\sum_{i=1}^{N} F_{y,i} = 0\right\}$$

There are two unknowns, F_A and F_B, and so an additional equation is needed. By summing moments about point B, we have

$$(70\ \text{N})(90\ \text{mm}) + F_A(20\ \text{mm}) = 0 \qquad \rightarrow \left\{\sum_{i=1}^{N} M_{B,i} = 0\right\}$$

The negative sign indicates that F_A produces a clockwise moment about point B. The cutting force is $F_A = 315$ N, and, after back substitution, we find that $F_B = 385$ N. Because these numerical values are each positive, the directions shown at the outset on the free body diagram are correct.

Discussion

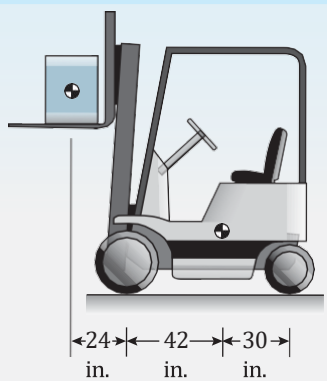
The wire cutters operate according to the principle of a lever. The cutting force is proportional to the force on the handles, and it is also related to the ratio of distances AB and BC. The mechanical advantage for a machine is defined as the ratio of the output and input forces, or in this case, (315 N)/(70 N) = 4.5. The wire cutters magnify the gripping force by 450%.

$$F_A = 315\ \text{N}$$
$$F_B = 385\ \text{N}$$

Example 4.7 Forklift Load Capacity

The forklift weighs 3500 lb and carries an 800-lb shipping container. There are two front wheels and two rear wheels on the forklift. (a) Determine the contact forces between the wheels and ground. (b) How heavy a load can be carried before the forklift will start to tip about its front wheels? (See Figure 4.21.)

Figure 4.21

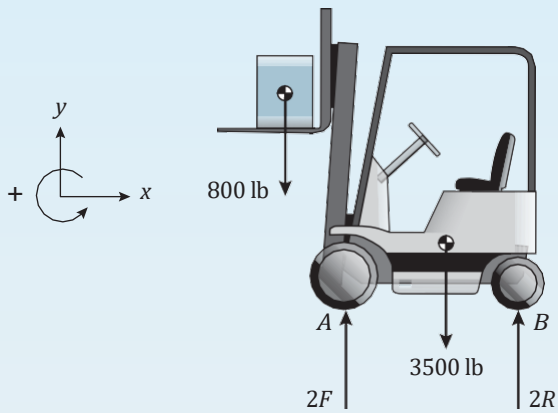


Example 4.7 *continued*

Approach

We are tasked with finding the wheel contact forces and the maximum load the forklift can carry before it tips forward. We assume that the forklift is not moving and then draw the free body diagram along with the positive sign conventions for forces and moments. We label the (known) 3500-lb and 800-lb weights that act through the mass centers of the forklift and container. The (unknown) force between a front wheel and the ground is denoted by F, and the (unknown) force between a rear wheel and the ground is R. On the side view of the free body diagram, the net effects of those forces on the wheel pairs are $2F$ and $2R$. (See Figure 4.22.)

Figure 4.22



Solution

(a) There are two unknowns (F and R), and therefore two independent equilibrium equations are needed to solve the problem. We first sum forces in the vertical direction

$$2(800\text{ lb}) + (3500\text{ lb}) + 2F + 2R = 0 \qquad +\uparrow \left\{\sum_{i=1}^{N} F_{y,i} = 0\right\}$$

or $F + R = 2150$ lb, but a second equation is required to determine the two unknowns. Summing forces in the horizontal direction will not provide any useful information; so we use a moment balance. Any location can be chosen as the pivot point. By choosing the point to coincide with the front wheel, force F will be conveniently eliminated from the calculation. Taking moments about point A, we have

$$(800\text{ lb})(24\text{ in.}) + (3500\text{ lb})(42\text{ in.}) + (2R)(72\text{ in.}) = 0 \qquad +\circlearrowleft \left\{\sum_{i=1}^{N} M_{A,i} = 0\right\}$$

Example 4.7 *continued*

from which $R = 888$ lb. Here, the 800-lb weight and the rear wheel forces exert counterclockwise (positive) moments about A, and the forklift's 3500-lb weight generates a negative moment. Substituting the solution for R into the vertical-force balance

$$F + (888 \text{ lb}) = 2150 \text{ lb}$$

returns $F = 1262$ lb.

(b) When the forklift is on the verge of tipping about the front wheels, the rear wheel has just lost contact with the ground, and $R = 0$. We denote the new weight of the shipping container that causes tipping as w. The moment balance about the front wheels is

$$(w)(24 \text{ in.}) - (3500 \text{ lb})(42 \text{ in.}) = 0 \qquad \circlearrowright \sum_{i=1}^{N} M_{A,i} = 0$$

The forklift will be on the verge of tipping when the operator attempts to lift a $w = 6125$-lb container.

Discussion

It makes sense that the front wheel forces are larger than the rear wheel, because the load is ahead of the front wheels. The sum of the wheel forces also equals the combined weight of the forklift and load. The large weight necessary to cause tipping is also expected as part of the forklift's design. As a double-check to our solution of part (a), we can determine the front wheel force without having to find the force carried by the rear wheels. The key is to sum moments about the rear wheels at point B. The unknown force R passes through that point, and, by having a perpendicular lever arm with zero length, it will be eliminated in the moment calculation. The moment balance now reads

$$(800 \text{ lb})(96 \text{ in.}) + (3500 \text{ lb})(30 \text{ in.}) - (2F)(72 \text{ in.}) = 0 \qquad \circlearrowright \sum_{i=1}^{N} M_{B,i} = 0$$

We obtain $F = 1262$ lb through only one equation and without any intermediate steps involving R. In short, by carefully choosing the moment's pivot point, we can reduce the amount of algebra by directly eliminating unknown forces.

Wheel forces: $F = 1262$ lb and $R = 888$ lb
Maximum tipping load: $w = 6125$ lb

4.6 Design Application: Rolling-Element Bearings

In the preceding sections, we discussed the properties of forces and moments and applied the requirements of equilibrium to examine the forces acting on structures and machines. We now consider a specific application to mechanical design and the forces that act on the machine components called *rolling-element bearings*. Bearings are used to hold shafts that rotate relative to fixed supports (for instance, the housing of a motor, gearbox, or transmission). In designing power transmission equipment, mechanical engineers will often perform a force or equilibrium analysis to choose the correct size and type of bearing for an particular application.

Bearings are classified into two broad groups: rolling contact and journal. In this section, rolling contact bearings are the focus, and each comprises the following components:

- An inner race
- An outer race
- Rolling elements in the form of balls, cylinders, or cones
- A separator that prevents the rolling elements from rubbing up against one another

Journal bearing

Rolling-element bearings are so common in machine design that they are found in applications as diverse as hard disk drives, bicycle wheels, robotic joints, and automobile transmissions. *Journal bearings*, on the other hand, have no rolling elements. Instead, the shaft simply rotates within a polished sleeve that is lubricated by oil or another fluid. Just as the puck on an air hockey table slides smoothly over a thin film of air, the shaft in a journal bearing slides over and is supported by a thin film of oil. Although they may be less familiar to you, journal bearings are also quite common, and they are used to support shafts in internal-combustion engines, pumps, and compressors.

A sample installation of a rolling contact bearing is shown in Figure 4.23. The shaft and the bearing's inner race rotate together, while the outer race and the case are stationary. When a shaft is supported in this way, the bearing's outer race will fit tightly into a matching circular recess that is formed in the case. As the shaft turns and transmits power, perhaps in a geartrain or transmission, the bearing could be subjected to forces oriented either along the shaft (a *thrust force*) or perpendicular to it (a *radial force*). An engineer will make a decision on the type of bearing to be used in a machine depending on whether thrust forces, radial forces, or some combination of the two are going to act on the bearing.

Thrust force
Radial force

Ball bearings

The most common type of rolling-element bearing is the *ball bearing*, which incorporates hardened, precision-ground steel spheres. Figure 4.24 depicts the major elements of a standard ball bearing: the inner race, outer race, balls, and separator. The *inner and outer races* are the bearing's connections to the shaft and the case. The *separator* (which is sometimes

Inner and outer races
Separator, cage, retainer

Figure 4.23

An installation of a ball bearing.

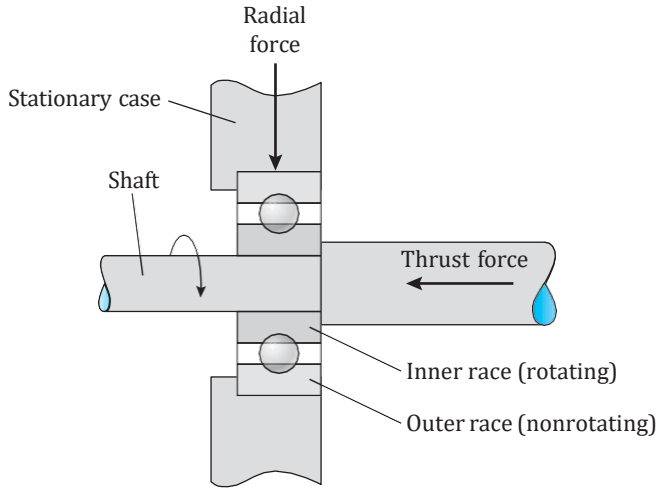
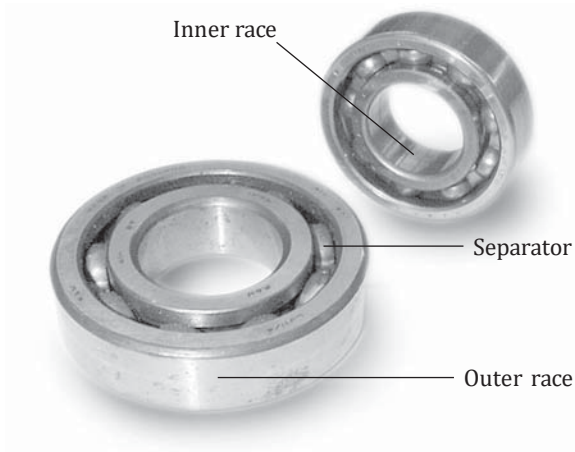


Figure 4.24

Elements of a ball bearing.

Image courtesy of the authors.



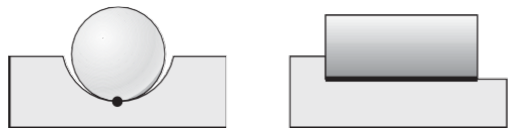
called the *cage* or *retainer*) keeps the balls evenly spaced around the bearing's perimeter and prevents them from contacting one another. Otherwise, if the bearing was used at high speed or subjected to large forces, friction could cause it to overheat and become damaged. In some cases, the gap between the inner and outer races is sealed by a rubber or plastic ring to keep grease in the bearing and dirt out of it.

Seals

In principle, the bearing's balls press against the inner and outer races at single points, similar to the contact between a marble and a floor. The force that each ball transfers between the inner and outer races therefore becomes concentrated on those surfaces in an intense and relatively sharp manner, as indicated in Figure 4.25(a) (see on page 160). If those forces instead could be spread over a larger area, we would expect that the bearing would wear less

Figure 4.25

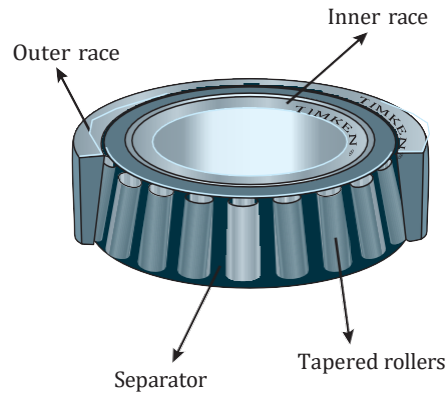
Side view of (a) point contact between a spherical ball and the raceway of a bearing, and (b) line contact occuring in a straight roller bearing.



(a) (b)



(a)



(b)

Figure 4.26 (a) A straight roller bearing. The inner race has been removed to show the rollers and separator. (b) A tapered roller bearing that is widely used in the front wheels of automobiles.

(a) Image courtesy of the authors. (b) Reprinted with permission by The Timken Company.

and last longer. With that in mind, rolling-element bearings that incorporate cylindrical rollers or tapered cones in place of spherical balls are one solution for distributing forces more evenly.

Straight roller bearings

Straight (or cylindrical) *roller bearings*, as shown in Figure 4.25(b), can be used to distribute forces more evenly over the bearing's races. If you place a few pens between your hands and then rub your hands together, you have the essence of a straight roller bearing. Figure 4.26(a) illustrates the structure of a straight roller bearing.

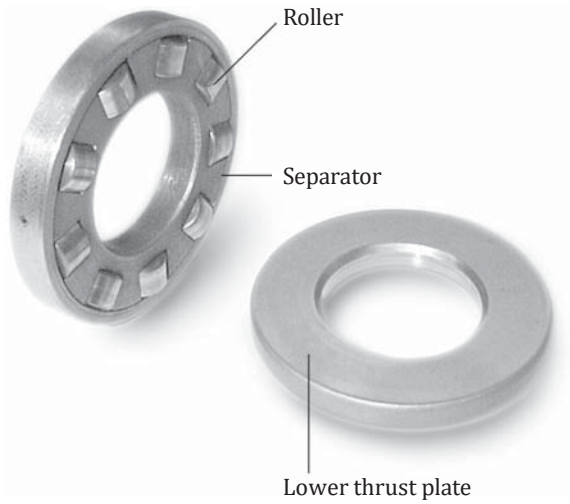
Tapered roller bearings

While straight roller bearings support forces that are directed mostly radially, *tapered* (or angled) *roller bearings* can support a combination of radial and thrust forces. This is because these bearings are built around rollers

Figure 4.27

A thrust roller bearing.

Image courtesy of the authors.



shaped like truncated cones [Figure 4.26(b)]. One prominent application for tapered roller bearings is in automobile wheel bearings because both radial forces (the weight of the vehicle) and thrust forces (the cornering force generated when the vehicle makes a turn) are present.

Thrust roller bearings

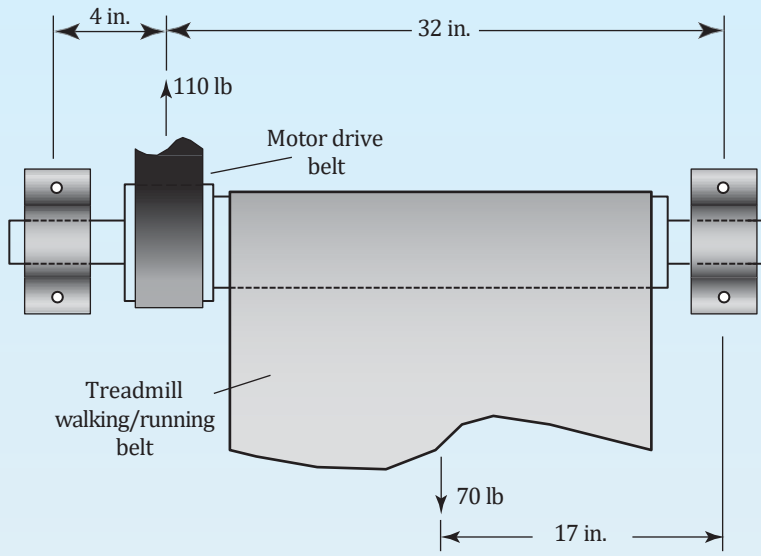
While straight roller bearings support forces that are directed mostly radially, and tapered roller bearings can support a combination of radial and thrust forces, the *thrust roller bearing* carries loads that are directed mostly along a shaft. One type of thrust bearing is shown in Figure 4.27. The rolling elements in this case are cylinders having a slight barrel shape to them. In contrast to the straight roller bearing of Figure 4.26(a), these rollers are oriented radially and perpendicular to the shaft. Thrust bearings are appropriate for such applications as a rotating table that must support the dead weight of cargo but that also needs to turn freely.

Example 4.8 Treadmill's Belt Drive

An electric motor is used to power an exercise treadmill. Forces are applied to the treadmill's shaft by the motor's drive belt and by the wide, flat belt that is the surface used for walking or running.The tight and loose spans of the drive belt together apply 110 lb to the shaft, and the treadmill's belt applies 70 lb. The shaft is supported by ball bearings on each side of the belt. Calculate the magnitudes and directions of the forces exerted by the shaft on the two bearings. (See Figure 4.28 on page 162.)

Example 4.8 *continued*

Figure 4.28



Approach

We are tasked with finding the forces on the two bearings from the two belts. We first assume that all the forces act parallel to the y-direction. The free body diagram of the shaft is drawn, along with the sign conventions for the coordinate directions and rotation. On the diagram, we first label the 110-lb and 70-lb belt tensions, and then we denote the forces exerted by the bearings on the shaft as F_A and F_B. At this point, we don't know whether those unknown forces act in the positive or negative y-directions. By drawing them on the free body diagram using our sign convention, we will rely on the calculation to determine the actual direction of the forces. If a numerical value turns out to be negative, the result will mean that the force acts in the negative y-direction. (See Figure 4.29.)

Solution

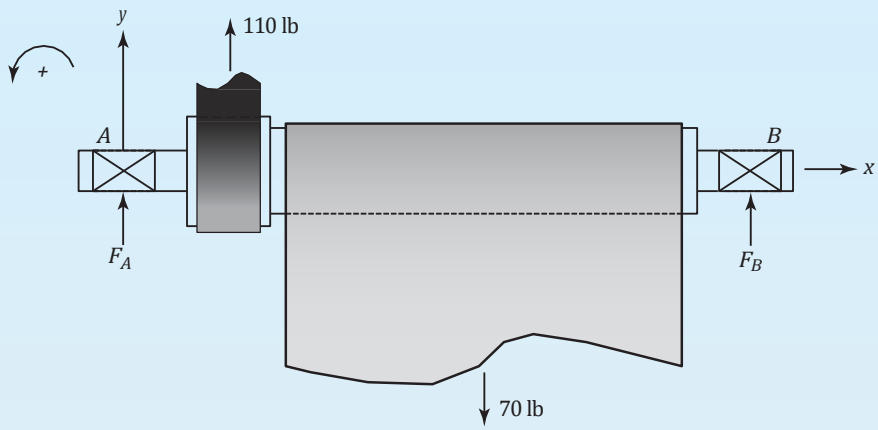
Because there are two unknowns (F_A and F_B), two equilibrium equations are needed to solve the problem. By summing forces in the y-direction

$$(110 \text{ lb}) + (70 \text{ lb}) + F_A + F_B = 0 \qquad \rightarrow \left\{ \sum_{i=1}^{N} F_{y,i} = 0 \right\}$$

or $F_A + F_B = 240$ lb. We will apply a moment balance for the second equation. By choosing the pivot point to coincide with the center of bearing A, force F_A will be eliminated from the calculation. Summing moments about

Example 4.8 *continued*

Figure 4.29



point A, we have

$$(110 \text{ lb})(4 \text{ in.}) + (70 \text{ lb})(19 \text{ in.}) - (F_B)(36 \text{ in.}) = 0 \qquad \rightarrow \left\{ \sum_{i=1}^{N} M_{A,i} = 0 \right\}$$

and $F_B = 24.72$ lb. The motor belt's tension and F_B exert counterclockwise (positive) moments about A, and the treadmill belt's tension balances those components with a negative moment. Substituting this value for F_B into the force balance returns $F_A = 264.72$ lb.

Discussion

These forces are the same order of magnitude as the applied forces, and that makes sense. Also, when in use, the treadmill will exert forces on the bearings in the x- and z-directions, but those are not accounted for in our analysis. Since the calculated value for F_A is negative, the force that is exerted by bearing A on the shaft acts in the negative y-direction with magnitude of 64.72 lb. Following the principle of action–reaction from Newton's third law, the directions of the forces exerted by the shaft on the bearings are opposite those exerted by the bearings on the shaft.

Bearing A: 64.72 lb in the negative y-direction

Bearing B: 24.72 lb in the positive y-direction

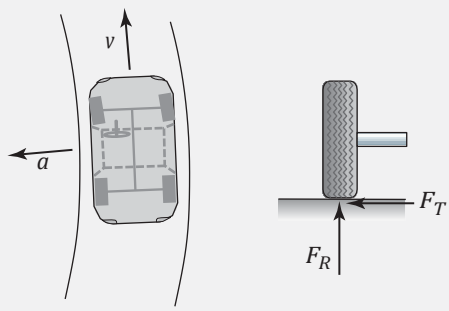
Example 4.9 Automobile Wheel Bearings

A 13.5-kN automobile is being driven at 50 km/h through a turn of radius of 60 m. Assuming that the forces are equally balanced among the four wheels, calculate the magnitude of the resultant force acting on the tapered roller bearings that support each wheel. To calculate the cornering force, apply Newton's second law ($F = ma$) with the centripetal acceleration ($a = v^2/r$) where m is the vehicle's mass, v denotes its speed, and r is the turn radius.

Approach

We are tasked with finding the resultant force from the applied radial and thrust forces on the vehicle wheel bearings. We assume that each wheel carries one-quarter of the vehicle's weight, and that force component is oriented radially on the wheel's bearings. The cornering force acting on the entire vehicle is mv^2/r, directed toward the center of the turn, and the fraction carried by each wheel is a thrust force parallel to the wheel's axle. In terms of those variables, we will determine a general symbolic equation for the magnitude of a wheel's resultant force, and then we will substitute specific values to obtain a numerical result. (See Figure 4.30.)

Figure 4.30



Solution

With w denoting the automobile's weight, each wheel carries the radial force

$$F_R = \frac{w}{4}$$

The thrust force carried by one wheel is given by the expression

$$F_T = \frac{1}{4}\frac{mv^2}{r} \qquad \rightarrow (F = ma)$$

where the vehicle's mass is

$$m = \frac{w}{g}$$

Example 4.9 *continued*

The resultant of those two perpendicular force components is

$$F = \sqrt{F_R^2 + F_T^2} \qquad \rightarrow \left\{F = \sqrt{F_x^2 + F_y^2}\right\}$$

$$= \frac{m}{4}\sqrt{g^2 + \left(\frac{v^2}{r}\right)^2}$$

Next, we will substitute the numerical values given in the problem's statement into this general expression. The vehicle's mass is

$$m = \frac{13.5 \times 10^3 \text{ N}}{9.81 \text{ m/s}^2} \qquad \rightarrow \left\{m = \frac{w}{g}\right\}$$

$$= 1.376 \times 10^3 \left(\frac{\text{kg} \cdot \text{m}}{\text{s}^2}\right)\left(\frac{\text{s}^2}{}\right)$$

$$= 1.376 \times 10^3 \text{ kg}$$

or 1.376 Mg. In consistent dimensions, the velocity is

$$v = \left(50 \frac{\text{km}}{\text{h}}\right)\left(10^3 \frac{\text{m}}{\text{km}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right)$$

$$= 13.89 \left(\frac{\text{km}}{\text{h}}\right)\left(\frac{\text{m}}{\text{km}}\right)\left(\frac{\text{h}}{\text{s}}\right)$$

$$= 13.89 \frac{\text{m}}{\text{s}}$$

With a 60-m turn radius, the magnitude of the resultant force acting on a wheel's bearings is

$$F = \left(\frac{1.376 \times 10^3 \text{ kg}}{4}\right)\sqrt{\left(9.81 \frac{\text{m}}{\text{s}^2}\right)^2 + \left(\frac{(13.89 \text{ m/s})^2}{60 \text{ m}}\right)^2} \qquad \rightarrow \left\{F = \frac{m}{4}\sqrt{g^2 + \left(\frac{v^2}{r}\right)^2}\right\}$$

$$= 3551 \frac{\text{kg} \cdot \text{m}}{\text{s}^2}$$

$$= 3551 \text{ N}$$

$$= 3.551 \text{ kN}$$

Discussion

Because of the cornering force, the wheel bearings carry somewhat more than one-quarter of the vehicle's weight (3.375 kN). As a double-check, we can verify the dimensional consistency of the calculations by noting that the term v^2/r, which is combined with the gravitational acceleration g, has the dimensions of acceleration.

$$F = 3.551 \text{ kN}$$

SUMMARY

The objective of this chapter has been to introduce the engineering concepts of force systems, moments, and equilibrium in the context of engineering structures and machines. The primary variables, symbols, and dimensions are summarized in Table 4.3, and the key equations are listed in Table 4.4. After developing these concepts, we applied them to determine the magnitudes and directions of forces acting on and within simple structures and machines. Engineers often perform a force analysis to see whether a design will be feasible and safe. One of the skills that mechanical engineers develop is the ability to apply equations to physical problems clearly and consistently. Selecting the object to be included in a free body diagram, choosing the directions for coordinate axes, and picking the best point for balancing moments are some of the choices that you need to make in solving problems of this nature.

We also applied the concepts of force systems to several different types of rolling element bearings that are used in machine design. Bearings and the other machine components, which we will examine in later chapters, have special properties and terminology, and mechanical engineers need to be fluent with those building blocks to select the component that is best suited for particular hardware.

In the next chapter, we will take another step toward designing structures and machines so that they will be strong enough to support the forces acting on them. We will build on the properties of force systems and take into account the strength characteristics of the materials from which the mechanical components are made.

Table 4.3 Quantities, Symbols, and Units that Arise when Analyzing Forces in Structures and Machines

		Conventional Units	
Quantity	**Conventional Symbols**	**USCS**	**SI**
Force vector	$\mathbf{F}$	lb	N
Force components	F_x, F_y, $F_{x,i}$, $F_{y,i}$	lb	N
Force magnitude	F	lb	N
Force direction	u	deg, rad	deg, rad
Resultant	$\mathbf{R}$, R, R_x, R_y	lb	N
Moment about O	M_o, $M_{o,i}$	in · lb, ft · lb	N · m
Perpendicular lever arm	d	in., ft	m
Moment component offset	Δx, Δy	in., ft	m

Table 4.4

Key Equations that Arise when Analyzing Forces in Structures and Machines

Force vector Rectangular-polar conversion Polar-rectangular conversion	$F = \sqrt{F_x^2 + F_y^2}, \quad \theta = \tan^{-1}\left(\frac{F_y}{F_x}\right)$ $F_x = F\cos\theta, \quad F_y = F\sin\theta$
Resultant of N forces	$R_x = \sum_{i=1}^{N} F_{x,i}, \quad R_y = \sum_{i=1}^{N} F_{y,i}$
Moment about point O Perpendicular lever arm Moment components	$M_o = Fd$ $M_o = -F_x \Delta y + F_y \Delta x$
Equilibrium Displacement Rotation	$\sum_{i=1}^{N} F_{x,i} = 0, \quad \sum_{i=1}^{N} F_{y,i} = 0$ $\sum_{i=1}^{N} M_{o,i} = 0$

Self-Study and Review

4.1. What are Newton's three laws of motion?

4.2. What are the conventional dimensions for forces and moments in the USCS and SI?

4.3. About how many newtons are equivalent to 1 lb?

4.4. How do you calculate the resultant of a force system by using the vector algebra and vector polygon methods? When do you think it is more expedient to use one method over the other?

4.5. How do you calculate a moment by using the perpendicular lever arm and moment components methods? When do you think it is more expedient to use one method over the other?

4.6. Why is a sign convention used when calculating moments using the component method?

4.7. What are the equilibrium requirements for particles and rigid bodies?

4.8. What steps are involved in drawing a free body diagram?

4.9. Describe some of the differences between ball, straight roller, tapered roller, and thrust bearings. Give examples of real situations in which you would select one to be used instead of another.

4.10. What is the purpose of a bearing's separator?

4.11. Make a cross-sectional drawing of a tapered roller bearing.

4.12. Give examples of situations where bearings are subjected to radial forces, thrust forces, or some combination of the two.

Materials and Stresses

- Identify circumstances in which a mechanical component is loaded in tension, compression, or shear, and calculate the stress present.
- Sketch a stress–strain curve, and use it to describe how a material responds to the loads applied to it.
- Explain the meaning of the material properties known as the elastic modulus and yield strength.
- Understand the differences between elastic and plastic responses of materials and between their ductile and brittle behaviors.
- Discuss some of the properties and uses for metals and their alloys, ceramics, polymers, and composite materials.
- Apply the concept of a factor of safety to the design of mechanical components that are subjected to tension or shear stress.

5.1 Overview

As one of their responsibilities, mechanical engineers design hardware so that it shouldn't break when being used and so that it can carry the forces acting on it reliably and safely. As an example, consider Boeing's 767 commercial airliner, which weighs up to 350,000 lb when fully loaded. When the airplane is parked on the ground, its weight is supported by the landing gear and wheels. During flight, the aircraft's wings create an upward lift force that exactly balances the weight. Each wing, therefore, carries a force that is equal to half of the airplane's weight, which in this case is equivalent to some 70 family-sized automobile sedans. Subjected to the lift force, the wings bend upward, and, if the flight happens to encounter rough weather, the wings will bend up and down by an additional considerable amount as the plane is buffeted by turbulence. When engineers selected the aircraft's materials, they took into account the facts that an aircraft's wings are subjected to large forces, that they sag under their self-weight, and that they bend upward in response to the lift forces. The wings are designed to be strong, safe, and reliable while being no heavier than necessary to meet the design requirements.

By applying the properties of force to systems as described in Chapter 4, you have seen how to calculate the magnitudes and directions of forces that act on certain structures and machines. Knowing those forces alone, however,

is not enough information to decide whether a certain piece of hardware will be strong enough and not fail in its task. By "fail" or "failure," we not only mean that the hardware will not break, but also that it will not stretch or bend so much as to become significantly distorted. A 5-kN force, for instance, might be large enough to break a small bolt or to bend a shaft so much that it would wobble and not spin smoothly. A larger-diameter shaft or one that is made from a higher-grade material, on the other hand, might very well be able to support that force without incurring any damage.

With those thoughts in mind, you can see that the circumstances for a mechanical component to break, stretch, or bend depend not only on the forces applied to it, but also on its dimensions and the properties of the material from which it is made. Those considerations give rise to the concept of *stress* as a measure of the intensity of a force applied over a certain area. Conversely, the *strength* of a material describes its ability to support and withstand the stress applied to it. Engineers compare the stress present in a component to the strength of its material in order to determine whether the design is satisfactory. As an example, the broken crankshaft shown in Figure 5.1 was removed from a single-cylinder internal-combustion engine. This failure was accelerated by the presence of sharp corners in the shaft's rectangular keyway, which is used to transfer torque between the shaft and a gear or pulley. The spiral shape of the fracture surface indicates that the shaft had been overloaded by a high torque prior to breaking. Engineers are able to combine their knowledge of forces, materials, and dimensions in order to learn from past failures and to improve and evolve the design of new hardware.

Stress
Strength

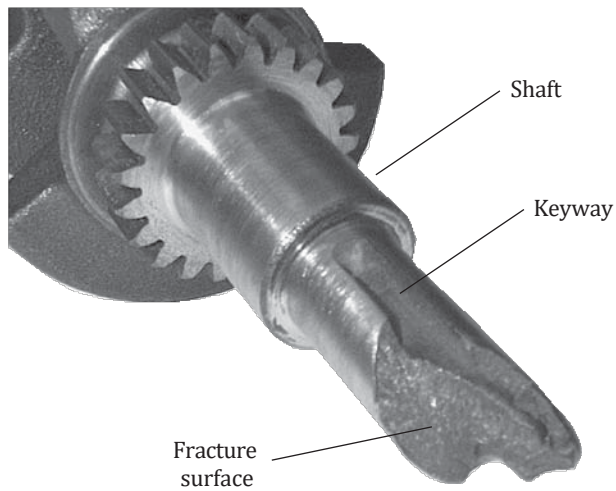
In this chapter, we will discuss some of the properties of engineering *materials* and examine the *stresses* that can develop within them. Within these topics lie the discipline known as solid mechanics, and they fit into the hierarchy of mechanical engineering topics shown in Figure 5.2. Tension,

Element 4: Materials and stresses

Figure 5.1

A broken crankshaft that was removed from a single-cylinder internal-combustion engine.

Image courtesy of the authors.



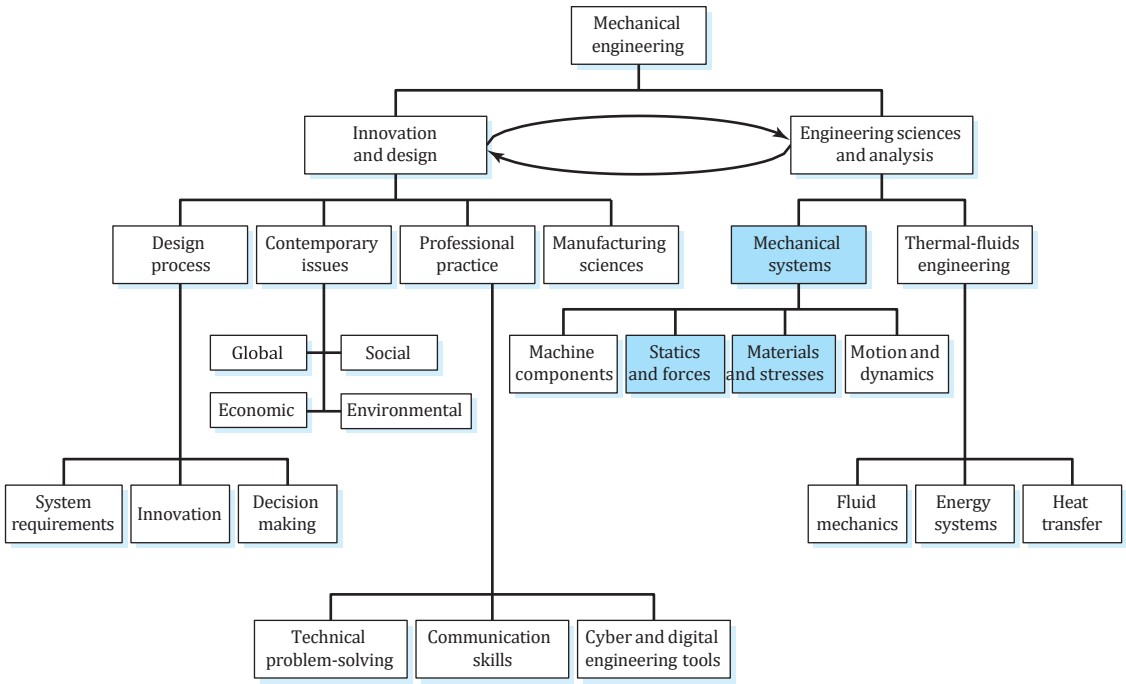


Figure 5.2 Relationship of the topics emphasized in this chapter (shaded boxes) relative to an overall program of study in mechanical engineering.

compression, and shear stress are quantities that engineers calculate when they relate the dimensions of a mechanical component to the forces acting on it. Those stresses are then compared to the material's physical properties to determine whether failure is expected to occur. When the strength exceeds the stress, we expect that the structure or machine component will be able to carry the forces without incurring damage. Engineers conduct these types of force, stress, materials, and failure analyses while they design products.

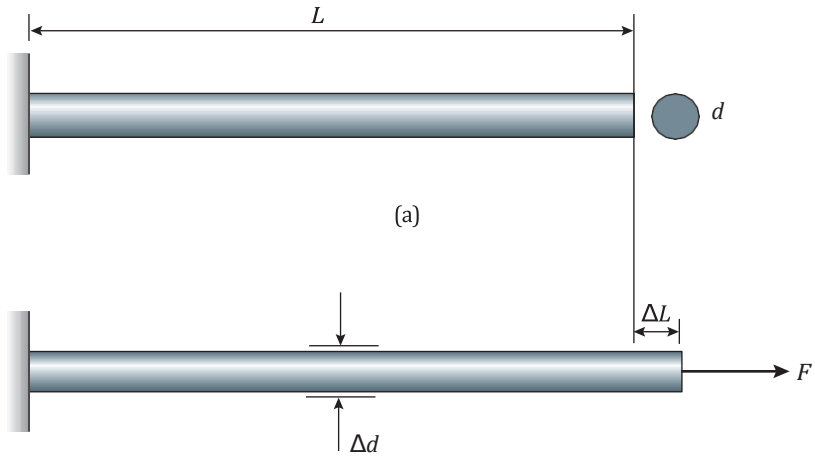
5.2 Tension and Compression

The type of stress that is most readily visualized and useful for you to develop intuition about materials and stresses is called tension and compression. Figure 5.3 (see on page 186) shows a built-in round rod that is held fixed at its left end and then later placed in tension by the force F that pulls on the right end. Before the force is applied, the rod has original length L, diameter d, and cross-sectional area

$$A = \pi \frac{d^2}{4} \tag{5.1}$$

Figure 5.3

A straight rod is stretched and placed in tension.



Engineers usually calculate the cross-sectional area of round rods, bolts, and shafts in terms of their diameter rather than their radius r ($A = \pi r^2$) since it's more practical to measure the diameter of a shaft using a caliper gauge. As the force F is gradually applied, the rod stretches along its length by the amount ΔL shown in Figure 5.3(b). In addition, the diameter of the rod shrinks by a small amount due to an effect known as *Poisson's contraction*, which is a topic described in the next section. In any event, the change in diameter Δd is smaller and usually less noticeable than the lengthwise stretch ΔL. To gauge the relative amounts of ΔL and Δd, try stretching a rubber band to notice how its length, width, and thickness change.

Poisson's contraction

If the force is not too great, the rod will return to its original diameter and length (just like a spring) when F is removed. If the rod is not permanently deformed after F has been applied, the stretching is said to occur *elastically.* Alternatively, the force could have been great enough that the rod would be *plastically* deformed, meaning that, when the force was applied and then removed, the rod would be longer than it was initially. You can experiment with a desktop paper clip to see firsthand the difference between the elastic and plastic behavior of materials. Bend one end of the paper clip a small amount—perhaps just a millimeter or two—and notice how it springs back to the original shape when you release it. On the other hand, you could unwrap the paper clip into a nearly straight piece of wire. In that case, the paper clip does not spring back. The force was large enough to permanently change the material's shape through plastic deformation.

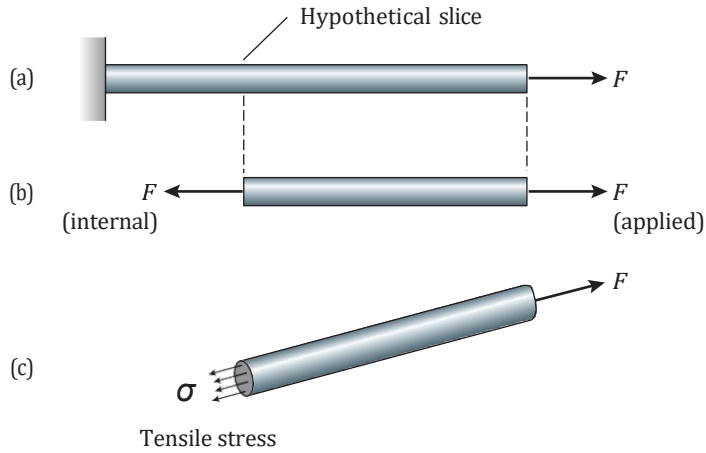
Elastic behavior

Plastic behavior

Although the force is applied at only one end of the rod in our example, its influence is felt at each cross section along the rod's length. As shown in Figure 5.4, imagine slicing through the rod at some interior point. The segment that is isolated in the free body diagram of Figure 5.4(b) shows F applied to

Figure 5.4

(a) A rod that has been stretched. (b) A section that is sliced from the rod to expose the internal force. (c) The tensile stress that is distributed over the rod's cross section.



Internal force

the rod's right-hand end and an equivalent *internal force* on the segment's left-hand end that balances *F*. Such must be the case; otherwise the segment shown in the free body diagram would not be in equilibrium. The location of our hypothetical slice through the rod is arbitrary, and we conclude that a force of magnitude *F* must be carried by the rod at each of its cross sections.

Since the rod is formed of a continuous solid material, we do not realistically expect that the internal force will be concentrated at a point as depicted by the force vector arrow in Figure 5.4(b). Instead, the influence of the force will be spread out and smeared over the rod's cross section; this process is the basic idea behind stresses in mechanical components. *Stress* is essentially an internal force that has been distributed over the area of the rod's cross section [Figure 5.4(c)], and it is defined by the equation

Stress

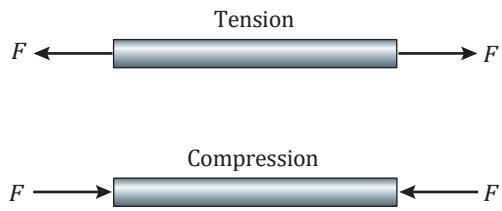
$$s = \frac{F}{A} \tag{5.2}$$

Like the force *F*, the stress σ (the lowercase Greek character sigma) is oriented perpendicularly to a hypothetical slice made through the cross section. When the stress tends to lengthen the rod, it is called *tension*, and $\sigma > 0$. On the other hand, when the rod is shortened, the stress is called *compression*. In that case, the direction of σ in Figure 5.4 reverses to point inward, and $\sigma < 0$. The orientations of forces in tension and compression are shown in Figure 5.5.

Tension
Compression

Figure 5.5

Directions of tensile and compressive forces.



Similar to the pressure within a liquid or gas, stress is also interpreted as a force that has been distributed over an area. Therefore, stress and pressure have the same dimensions. In the SI, the derived unit of stress is the pascal ($1\ \text{Pa} = 1\ \text{N/m}^2$), and the dimension pound-per-square-inch ($1\ \text{psi} = 1\ \text{lb/in}^2$) is used in the USCS. Because large numerical quantities frequently arise in calculations involving stresses and material properties, the prefixes "kilo" (k), "mega" (M), and "giga" (G) are applied to represent factors of 10^3, 10^6, and 10^9, respectively. Therefore,

$$1\ \text{kPa} = 10^3\ \text{Pa} \qquad 1\ \text{MPa} = 10^6\ \text{Pa} \qquad 1\ \text{GPa} = 10^9\ \text{Pa}$$

Despite mixing formats with the SI, it is also conventional to apply the prefixes "kilo" and "mega" when representing large numerical values for stress in the USCS. Mechanical engineers abbreviate 1000 psi as 1 ksi (without the "p"), and 1,000,000 psi as 1 Mpsi (with the "p"):

$$1\ \text{ksi} = 10^3\ \text{psi} \qquad 1\ \text{Mpsi} = 10^6\ \text{psi}$$

One billion psi is an unrealistically large value for USCS calculations involving materials and stresses in mechanical engineering applications, and so it is not used. Numerical values for stresses can be converted between the USCS and SI using the factors listed in Table 5.1. Referring to the table's first row, for instance, we see that

$$1\ \text{psi} = 10^{-3}\ \text{ksi} = 6.895 \times 10^3\ \text{Pa} = 6.895\ \text{kPa} = 6.895 \times 10^{-3}\ \text{MPa}$$

While stress is related to the intensity of a force's application, the engineering quantity called strain measures the amount that the rod stretches.

Elongation

The *elongation* ΔL in Figure 5.3 is one way to describe how the rod lengthens when F is applied, but it is not the only way, nor is it necessarily the best. If a second rod has the same cross-sectional area but is only half as long, then according to Equation (5.2), the stress within it would be the same as in the first rod. However, we expect intuitively that the shorter rod would stretch by a smaller amount. To convince yourself of this principle, hang a weight from two different lengths of rubber band, and notice how the longer band stretches more. Just as stress is a measure of force per unit area, the quantity called *strain* is defined as the amount of elongation that occurs per unit of

Strain

Table 5.1 Conversion Factors between USCS and SI Units for Stress

psi	ksi	Pa	kPa	MPa
1	10^{-3}	6.895×10^3	6.895	6.895×10^{-3}
10^3	1	6.895×10^6	6.895×10^3	6.895
1.450×10^{-4}	1.450×10^{-7}	1	10^{-3}	10^{-6}
0.1450	$1.450\ 3\ 10^{-4}$	10^3	1	10^{-3}
145.0	0.1450	10^6	10^3	1

the rod's original length. Strain ε (the lowercase Greek character epsilon) is calculated from the expression

$$\varepsilon = \frac{\Delta L}{L} \tag{5.3}$$

Because the length dimensions cancel out in the numerator and denominator, ε is a dimensionless quantity. Strain is generally very small, and it can be expressed either as a decimal quantity (for instance, $\varepsilon = 0.005$) or as a percentage ($\varepsilon = 0.5\%$).

Example 5.1 Hanger Rod in the Kansas City Hyatt Regency Hotel's Skyway

During the investigation of the collapse involving the hotel's elevated walkways (page 150), the connection between the hanger rod and the upper walkway was found to have broken at a load of approximately 20,500 lb. In the dimensions of ksi, determine the stress in the 1.25-in.-diameter hanger rod when the tension was 20,500 lb.

Approach

The hanger rod carries a tension force in the manner shown in Figure 5.3 with $F = 2.05 \times 10^4$ lb and $d = 1.25$ in. We are to calculate the stress by applying Equation (5.2).

Solution

By using Equation (5.1), the cross-sectional area of the hanger rod is

$$A = \frac{w(1.25\ \text{in.})^2}{4} \qquad \rightarrow \left\{A = \frac{wd^2}{4}\right\}$$
$$= 1.227\ \text{in}^2$$

The tensile stress is

$$\sigma = \frac{2.05 \times 10^4\ \text{lb}}{1.227\ \text{in}^2} \qquad \rightarrow \left\{\sigma = \frac{F}{A}\right\}$$
$$= 1.670 \times 10^4\ \frac{\text{lb}}{\text{in}^2}$$
$$= 1.670 \times 10^4\ \text{psi}$$

where we have substituted the definition of the derived unit psi (pound per square inch) for stress from Table 2.5. It is conventional to apply the abbreviation "ksi" to represent the factor of 1000 psi in a more compact form:

$$\sigma = (1.670 \times 10^4\ \text{psi})\left(10^{-3}\ \frac{\text{ksi}}{\text{psi}}\right)$$
$$= 16.70\ (\cancel{\text{psi}})\left(\frac{\text{ksi}}{\cancel{\text{psi}}}\right)$$
$$= 16.70\ \text{ksi}$$

Example 5.1 *continued*

Discussion

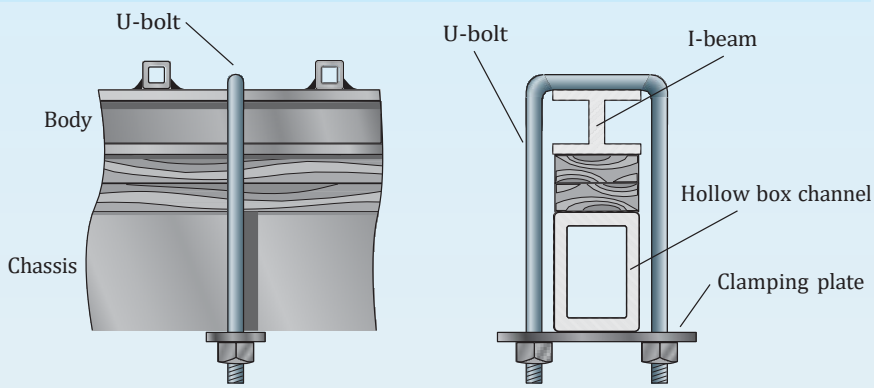
As we'll see in the following section, this level of stress is not particularly high when compared to the strength of steel materials. Although the tension may not have been sufficient to break the hanger rod, it was great enough to destroy the connection between the rod and the pedestrian walkway. As was the case in the skyways' collapse, it is not an uncommon situation in mechanical engineering for the connections between components to be weaker than the components themselves.

$$\sigma = 16.70 \text{ ksi}$$

Example 5.2 U-Bolt Clamp

The U-bolt is used to attach the body (formed with I-beam construction) of a commercial moving van to its chassis (formed from hollow box channel). (See Figure 5.6.) The U-bolt is made from a 10-mm-diameter rod, and the nuts on it are tightened until the tension in each straight section of the U-bolt is 4 kN. (a) Show how forces are transferred through this assembly by drawing free body diagrams of the U-bolt and its nuts, the body and chassis stack, and the clamping plate. (b) In the units of MPa, calculate the tensile stress in a straight section of the U-bolt.

Figure 5.6



Approach

In part (a), we will isolate three free bodies, and each must be in equilibrium: the U-bolt and its nuts, the body and chassis stack, and the clamping plate. The forces acting within the assembly will be equal in magnitude but

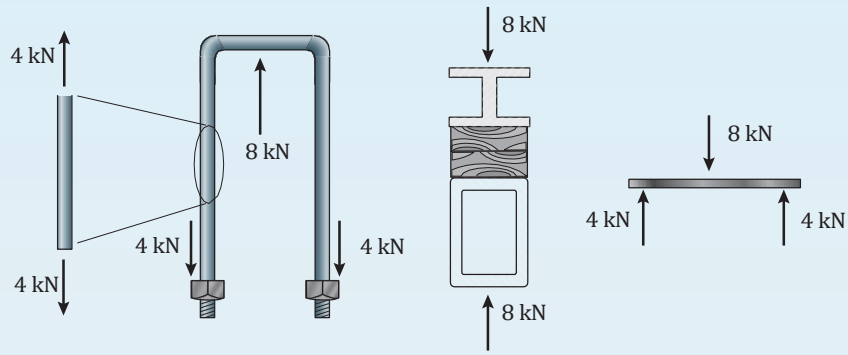
Example 5.2 *continued*

oppositely directed on adjacent components. The straight sections of the U-bolt are placed in tension in the manner shown in Figure 5.3 with $F = 4$ kN and $d = 10$ mm. We will calculate the stress by using Equation (5.2).

Solution

(a) Because each of the two straight sections of the U-bolt carries 4 kN of tension, their 8-kN resultant is transferred as compression to the body and chassis stack. (See Figure 5.7.) The 8-kN load is likewise applied by the box channel to the clamping plate. The equal and oppositely directed 4-kN forces act between the clamping plate and the nuts threaded onto the U-bolt.

Figure 5.7



Straight section U-bolt and nuts Body and chassis stack Clamping plate

(b) The cross-sectional area of the bolt is

$$A = \frac{\pi (10 \text{ mm})^2}{4} \rightarrow A = \frac{\pi d^2}{4}$$

$$= 78.54 \text{ mm}^2$$

which must be converted to dimensionally consistent units for stress in the SI:

$$A = (78.54 \text{ mm}^2)\left(10^{-3}\frac{\text{m}}{\text{m}}\right)^2$$

$$= 7.854 \times 10^{-5} (\text{mm}^2)\left(\frac{\text{m}^2}{\text{mm}^2}\right)$$

$$= 7.854 \times 10^{-5} \text{ m}^2$$

Example 5.2 *continued*

The tensile stress becomes

$$\sigma = \frac{4000\ \text{N}}{7.854 \times 10^{-5}\ \text{m}^2} \quad \rightarrow \left\{\sigma = \frac{F}{A}\right\}$$
$$= 5.093 \times 10^{7}\ \frac{\text{N}}{\text{m}^2}$$
$$= 5.093 \times 10^{7}\ \text{Pa}$$

where we have applied the definition of the derived unit pascal from Table 3.2. The SI prefix "mega" (Tables 3.3 and 5.1) condenses the large power-of-ten exponent in order to write the result in a more conventional form:

$$\sigma = (5.093 \times 10^{7}\ \text{Pa})\left(10^{-6}\ \frac{\text{MPa}}{\text{P}}\right)$$
$$= 50.93\ (\cancel{\text{Pa}})\left(\frac{\text{MPa}}{\cancel{\text{P}}}\right)$$
$$= 50.93\ \text{MPa}$$

Discussion

This is a significant tensile stress in the rod's straight sections; this makes sense because the applied load is large. In the upper portion of the U-bolt where the rod has 90° corners and is in contact with the I-beam, a more complicated state of stress is present, which would require a different analysis.

$$\sigma = 50.93\ \text{MPa}$$

Focus On MASDAR CITY

Masdar City has been designed to be one of the world's most sustainable urban environments (Figure 5.8.). Located 17 km outside Abu Dhabi, Masdar City is designed to be carbon neutral with no carbon emissions of any kind. Critical to the city's design is an infrastructure of streets, walkways, and buildings that are naturally cooled by the innovative structural design, solar shades, and wind towers. There will be no automobiles in the city. Rather, the residents will use an underground personal rapid transit system powered by solar-powered batteries. During the design of the city, engineers had to account for a wide range of loading conditions, forces, stresses, and material behaviors. As part of this design process, engineers had to select the best structural